Examples
Quadratic Functions – Part 1

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define the *general form* of a quadratic function.
2. Recognize the characteristics of the graphs of quadratic functions.
3. Given a quadratic function in general form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
4. Graph quadratic functions in *general form* by hand.

Example 1: Graph a Quadratic Function (1 of 6)

Graph the quadratic function $k(x) = x^2 - 4x - 5$ by hand.

Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens up.

Step 2 - Find the coordinates of the vertex,

The x-coordinate of the vertex using $-\frac{b}{2a}$.

In the given function, $a = 1$ and $b = -4$. Then the x-coordinate is $-\frac{-4}{2(1)} = 2$.

The y-coordinate of the vertex using $k\left(-\frac{b}{2a}\right)$.

$k(1) = (2)^2 - 4(2) - 5 = -9$. The y-coordinate is -9.

The coordinates of the vertex are $(2, -9)$.

Example 1: Graph a Quadratic Function (2 of 6)

Step 3 - If possible, find the point(s) associated with the *x*-intercept(s).

We must set $k(x) = y$ equal to 0!

$0 = x^2 - 4x - 5$

We are going to use factoring and the *Zero Product Principle* to solve this equation.

$(x - 5)(x + 1) = 0$

We set both factors equal to 0 as follows:

$x - 5 = 0$ and $x + 1 = 0$, then $x = 5$ and $x = -1$

There are two *x*-intercepts, and the coordinates of the points associated with them are $(5, 0)$ and $(-1, 0)$.

Example 1: Graph a Quadratic Function (3 of 6)

Step 4 - Find the point associated with the *y*-intercept.

We must set *x* equal to 0!

$k(0) = 0^2 – 4(0) – 5 = – 5$. This is the *y*-intercept.

The coordinates of the point associated with the *y*-intercept are (0, – 5).

Step 5 – Find the equation of the axis of symmetry. Use $x = -\frac{b}{2a}$.

The equation of the axis of symmetry is $x = -\frac{-4}{2(1)} = 2$.

Example 1: Graph a Quadratic Function (4 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are $(2, -9)$.
- The coordinates of the points associated with the *x*-intercepts are $(5, 0)$ and $(-1, 0)$.
- The coordinates of the point associated with the *y*-intercept are $(0, -5)$.
- The axis of symmetry is a vertical line passing through the vertex.

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape.

Example 1: Graph a Quadratic Function (5 of 6)

Step 6 continued

We are going to pick $x = -2, 1, 3, 4,$ and 6 and find the corresponding y-values.

$k(-2) = (-2)^2 - 4(-2) - 5 = 7$

$k(1) = (1)^2 - 4(1) - 5 = -8$

$k(3) = (3)^2 - 4(3) - 5 = -8$

$k(4) = (4)^2 - 4(4) - 5 = -5$

$k(6) = (6)^2 - 4(6) - 5 = 7$

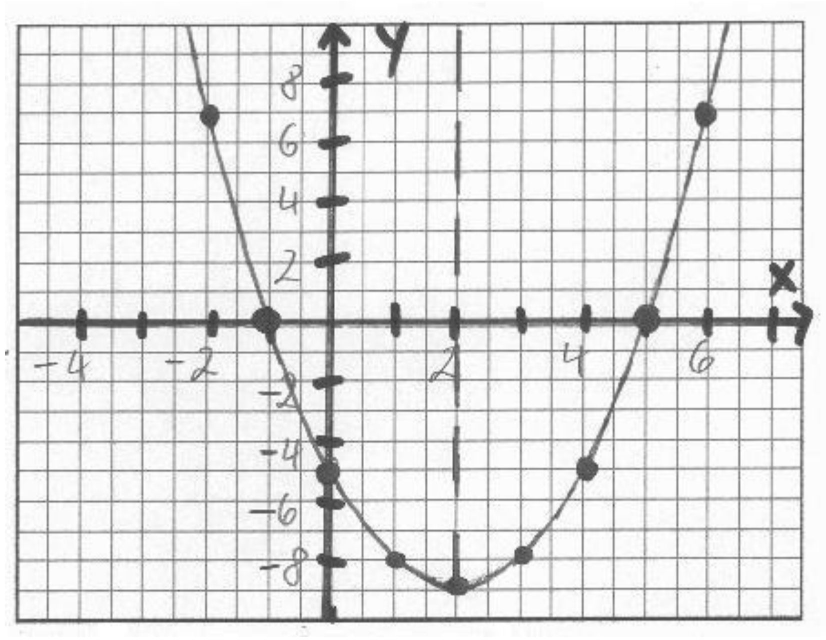
Example 1: Graph a Quadratic Function (6 of 6)

Step 6 continued

The coordinates of the additional points are

(– 2, 7), (1, – 8), (3, – 8), (4, – 5), and (6, 7)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!



Example 2: Graph a Quadratic Function (1 of 6)

Graph the quadratic function $g(x) = x^2 - 4x + 5$ by hand.

Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens up.

Step 2 - Find the coordinates of the vertex.

The x-coordinate of the vertex using $-\frac{b}{2a}$.

In the given function, $a = 1$ and $b = -4$. Then the x-coordinate is $-\frac{-4}{2(1)} = 2$.

The y-coordinate of the vertex using $g\left(-\frac{b}{2a}\right)$.

$g(2) = (2)^2 - 4(2) + 5 = 1$. The y-coordinate is 1.

The coordinates of the vertex are (2, 1).

Example 2: Graph a Quadratic Function (2 of 6)

Step 3 - If possible, find the point(s) associated with the *x*-intercept(s).

We must set $g(x) = y$ equal to 0!

Given $0 = x^2 - 4x + 5$, we are going to use the *Quadratic Formula* to solve this equation.

In the given function, $a = 1$, $b = -4$, and $c = 5$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

Reminder – Quadratic Formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that the radicand is a negative number. Paired with a square root, we know that we are encountering imaginary numbers.

Example 2: Graph a Quadratic Function (3 of 6)

Step 3 continued

Without any further simplifications, we can conclude that there are NO *x*-intercepts and move on to Step 4.

Step 4 - Find the point associated with the *y*-intercept.

We must set *x* equal to 0!

$g(0) = 0^2 – 4(0) + 5 = 5$. This is the *y*-intercept.

The coordinates of the point associated with the *y*-intercept are (0, 5).

Step 5 – Find the equation of the axis of symmetry. Use $x = -\frac{b}{2a}$.

The equation of the axis of symmetry is $x = -\frac{-4}{2(1)} = 2$.

Example 2: Graph a Quadratic Function (4 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (2, 1).
- There are no *x*-intercepts.
- The coordinates of the point associated with the *y*-intercept are (0, 5).
- The axis of symmetry is a vertical line passing through the vertex.

We really do not have enough points to make an adequate graph of the function. Therefore, let’s find a few more points close to the vertex so that we can draw a nice bowl shape.

Example 2: Graph a Quadratic Function

Step 6 continued

We are going to pick $x = -1, 1, 3, 4$, and 5 and find the corresponding y-values.

$g(-1) = (-1)^2 - 4(-1) + 5 = 10$
$g(1) = (1)^2 - 4(1) + 5 = 2$
$g(3) = (3)^2 - 4(3) + 5 = 2$
$g(4) = (4)^2 - 4(4) + 5 = 5$
$g(5) = (5)^2 - 4(5) + 5 = 10$

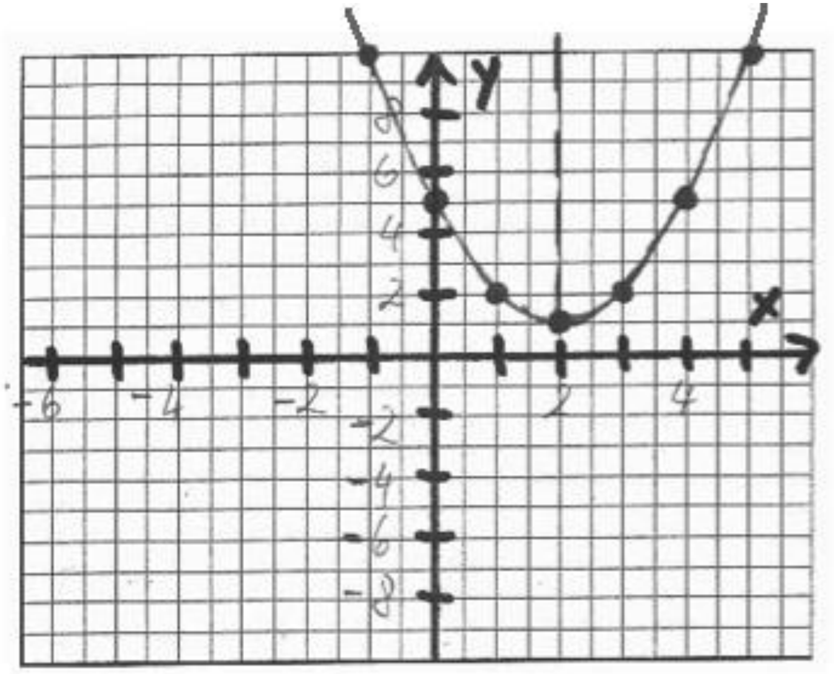
Example 2: Graph a Quadratic Function (6 of 6)

Step 6 continued

The coordinates of the additional points are

(– 1, 10), (1, 2), (3, 2), (4, 5), and (5, 10)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!



Example 3: Graph a Quadratic Function (1 of 6)

Graph the quadratic function $h(x) = x^2 - 2x + 1$ by hand.

Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens up.

Step 2 - Find the coordinates of the vertex.

The x-coordinate of the vertex using $-\frac{b}{2a}$.

In the given function, $a = 1$ and $b = -2$. Then the x-coordinate is $\frac{-(-2)}{2(1)} = 1$.

The y-coordinate of the vertex using $h\left(-\frac{b}{2a}\right)$.

$h(1) = (1)^2 - 2(1) + 1 = 0$. The y-coordinate is 0.

The coordinates of the vertex are (1, 0).

Example 3: Graph a Quadratic Function (2 of 6)

Step 3 - If possible, find the point(s) associated with the x-intercept(s)

We must set $h(x) = y$ equal to 0!

Given $0 = x^2 - 2x + 1$, we are going to use factoring and the *Zero Product Principle* to solve this equation.

$(x - 1)(x - 1) = 0$

Since both factors are the same, we only need to set one equal to 0 as follows:
$x - 1 = 0$

Then $x = 1$. This is the x-intercept.

There is only one x-intercept, and the coordinates of the point associated with it are (1, 0). Incidentally, this is also the coordinate of the vertex point!

Example 3: Graph a Quadratic Function (3 of 6)

Step 4 - Find the point associated with the *y*-intercept.

We must set x equal to 0!

$h(0) = 0^2 - 2(0) + 1 = 1$. This is the *y*-intercept.

The coordinates of the point associated with the *y*-intercept are (0, 1).

Step 5 – Find the equation of the axis of symmetry. Use $x = -\frac{b}{2a}$.

The equation of the axis of symmetry is $x = \frac{-(-2)}{2(1)} = 1$.

Example 3: Graph a Quadratic Function (4 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (1, 0).
- The coordinates of the point associated with the *x*-intercept are (1, 0)
- The coordinates of the point associated with the *y*-intercept are (0, 1).
- The axis of symmetry is a vertical line passing through the vertex.

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape.

Example 3: Graph a Quadratic Function

Step 6 continued

We are going to pick $x = -2, -1, 2, 3, 4$, and 5 and find the corresponding y-values.

$h(-2) = (-2)^2 - 2(-2) + 1 = 9$

$h(-1) = (-1)^2 - 2(-1) + 1 = 4$

$h(2) = (2)^2 - 2(2) + 1 = 1$

$h(3) = (3)^2 - 2(3) + 1 = 4$

$h(4) = (4)^2 - 2(4) + 1 = 9$

Example 3: Graph a Quadratic Function

Step 6 continued

The coordinates of the additional points are

(– 2, 9), (– 1, 4), (2, 1), (3, 4), and (4, 9)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!