Examples
Quadratic Functions

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define the general form of a quadratic function.
2. Define the standard form of a quadratic function.
3. Recognize the characteristics of the graphs of quadratic functions.
4. Given a quadratic function in general form and in standard form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
5. Graph quadratic functions by hand.

Example 1: Graph a Quadratic Function in General Form (1 of 6)

Graph the quadratic function $k(x) = x^2 - 4x - 5$ by hand.

Step 1 - Determine how the parabola opens.



$a = 1$, $a > 0$; the parabola opens up.

Step 2 - Find the coordinates of the vertex,

The x-coordinate of the vertex using $-\frac{b}{2a}$.

In the given function, $a = 1$ and $b = -4$. Then the x-coordinate is $-\frac{-4}{2(1)} = 2$.

The y-coordinate of the vertex using $k\left(-\frac{b}{2a}\right)$.

$k(1) = (2)^2 - 4(2) - 5 = -9$. The y-coordinate is -9.

The coordinates of the vertex are $(2, -9)$.

Example 1: Graph a Quadratic Function in General Form (2 of 6)

Step 3 - If possible, find the point(s) associated with the *x*-intercept(s).

We must set $k(x) = y$ equal to 0!

$0 = x^2 - 4x - 5$

We are going to use factoring and the *Zero Product Principle* to solve this equation.

$(x - 5)(x + 1) = 0$

We set both factors equal to 0 as follows:

$x - 5 = 0$ and $x + 1 = 0$, then $x = 5$ and $x = -1$

There are two *x*-intercepts, and the coordinates of the points associated with them are (5, 0) and (– 1, 0).

Example 1: Graph a Quadratic Function in General Form (3 of 6)

Step 4 - Find the point associated with the *y*-intercept.

We must set *x* equal to 0!

$k(0) = 0^2 - 4(0) - 5 = -5$. This is the *y*-intercept.

The coordinates of the point associated with the *y*-intercept are (0, – 5).

Step 5 – Find the equation of the axis of symmetry. Use $x = -\frac{b}{2a}$.

The equation of the axis of symmetry is $x = -\frac{-4}{2(1)} = 2$.

Example 1: Graph a Quadratic Function in General Form (4 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (2, – 9).
- The coordinates of the points associated with the *x*-intercepts are (5, 0) and (– 1, 0).
- The coordinates of the point associated with the *y*-intercept are (0, – 5).
- The axis of symmetry is a vertical line passing through the vertex.

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape.

Example 1: Graph a Quadratic Function in General Form (5 of 6)

Step 6 continued

We are going to pick $x = -2, 1, 3, 4$, and 6 and find the corresponding y-values.

$k(-2) = (-2)^2 - 4(-2) - 5 = 7$

$k(1) = (1)^2 - 4(1) - 5 = -8$

$k(3) = (3)^2 - 4(3) - 5 = -8$

$k(4) = (4)^2 - 4(4) - 5 = -5$

$k(6) = (6)^2 - 4(6) - 5 = 7$

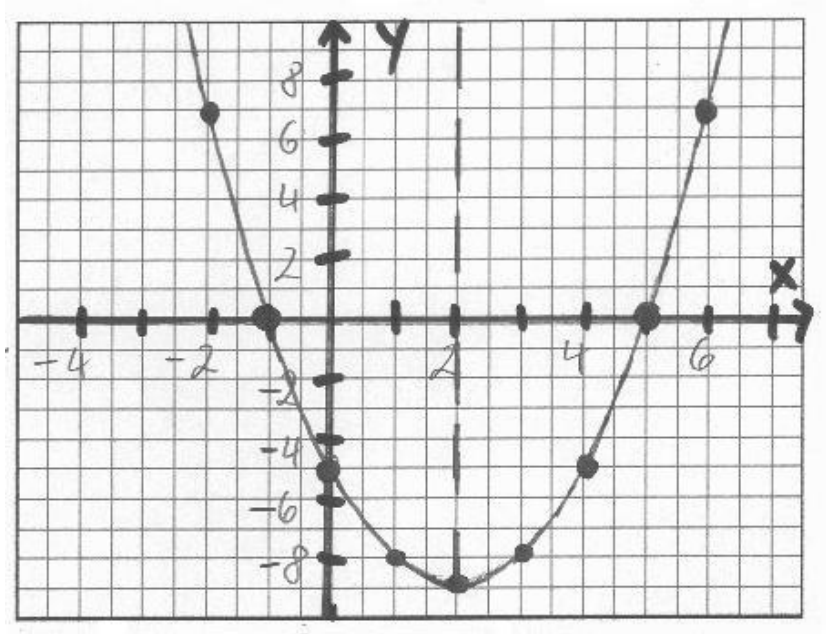
Example 1: Graph a Quadratic Function in General Form (6 of 6)

Step 6 continued

The coordinates of the additional points are

(– 2, 7), (1, – 8), (3, – 8), (4, – 5), and (6, 7)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!



Example 2: Graph a Quadratic Function in General Form (1 of 6)

Graph the quadratic function $g(x) = x^2 - 4x + 5$ by hand.



Step 1 - Determine how the parabola opens.

$a = 1$, $a > 0$; the parabola opens up.

Step 2 - Find the coordinates of the vertex.

The x-coordinate of the vertex using $-\frac{b}{2a}$.

In the given function, $a = 1$ and $b = -4$. Then the x-coordinate is $-\frac{-4}{2(1)} = 2$.

The y-coordinate of the vertex using $g\left(-\frac{b}{2a}\right)$.

$g(2) = (2)^2 - 4(2) + 5 = 1$. The y-coordinate is 1.

The coordinates of the vertex are (2, 1).

Example 2: Graph a Quadratic Function in General Form (2 of 6)

Step 3 - If possible, find the point(s) associated with the *x*-intercept(s).

We must set $g(x) = y$ equal to 0!

Given $0 = x^2 - 4x + 5$, we are going to use the *Quadratic Formula* to solve this equation.

In the given function, $a = 1$, $b = -4$, and $c = 5$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

Reminder – Quadratic Formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that the radicand is a negative number. Paired with a square root, we know that we are encountering imaginary numbers.

Example 2: Graph a Quadratic Function in General Form (3 of 6)

Step 3 continued

Without any further simplifications, we can conclude that there are NO *x*-intercepts and move on to Step 4.

Step 4 - Find the point associated with the *y*-intercept.

We must set *x* equal to 0!

$g(0) = 0^2 - 4(0) + 5 = 5$. This is the *y*-intercept.

The coordinates of the point associated with the *y*-intercept are (0, 5).

Step 5 – Find the equation of the axis of symmetry. Use $x = -\frac{b}{2a}$.

The equation of the axis of symmetry is $x = -\frac{-4}{2(1)} = 2$.

Example 2: Graph a Quadratic Function in General Form (4 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (2, 1).
- There are no *x*-intercepts.
- The coordinates of the point associated with the *y*-intercept are (0, 5).
- The axis of symmetry is a vertical line passing through the vertex.

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points close to the vertex so that we can draw a nice bowl shape.

Example 2: Graph a Quadratic Function in General Form (5 of 6)

Step 6 continued

We are going to pick $x = -1, 1, 3, 4$, and 5 and find the corresponding y-values.

$g(-1) = (-1)^2 - 4(-1) + 5 = 10$
$g(1) = (1)^2 - 4(1) + 5 = 2$
$g(3) = (3)^2 - 4(3) + 5 = 2$
$g(4) = (4)^2 - 4(4) + 5 = 5$
$g(5) = (5)^2 - 4(5) + 5 = 10$

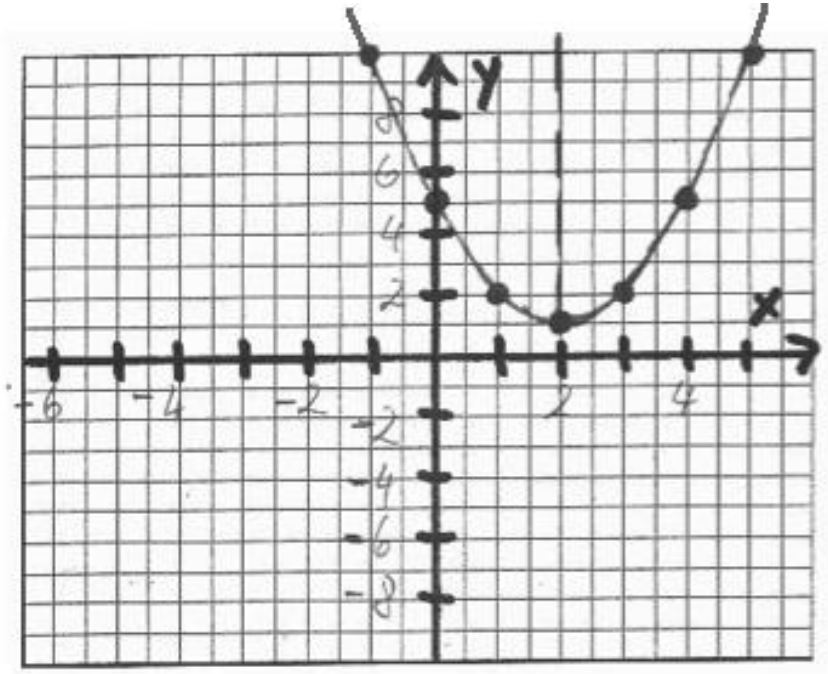
Example 2: Graph a Quadratic Function in General Form (6 of 6)

Step 6 continued

The coordinates of the additional points are

(– 1, 10), (1, 2), (3, 2), (4, 5), and (5, 10)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!



Example 3: Graph a Quadratic Function in General Form (1 of 6)

Graph the quadratic function $h(x) = x^2 - 2x + 1$ by hand.

Step 1 - Determine how the parabola opens.



$a = 1$, $a > 0$; the parabola opens up.

Step 2 - Find the coordinates of the vertex.

The x-coordinate of the vertex using $-\frac{b}{2a}$.

In the given function, $a = 1$ and $b = -2$. Then the x-coordinate is $\frac{-(-2)}{2(1)} = 1$.

The y-coordinate of the vertex using $h\left(-\frac{b}{2a}\right)$.

$h(1) = (1)^2 - 2(1) + 1 = 0$. The y-coordinate is 0.

The coordinates of the vertex are (1, 0).

Example 3: Graph a Quadratic Function in General Form (2 of 6)

Step 3 - If possible, find the point(s) associated with the x-intercept(s)

We must set $h(x) = y$ equal to 0!

Given $0 = x^2 - 2x + 1$, we are going to use factoring and the *Zero Product Principle* to solve this equation.

$(x - 1)(x - 1) = 0$

Since both factors are the same, we only need to set one equal to 0 as follows:
$x - 1 = 0$

Then $x = 1$. This is the x-intercept.

There is only one x-intercept, and the coordinates of the point associated with it are (1, 0). Incidentally, this is also the coordinate of the vertex point!

Example 3: Graph a Quadratic Function in General Form (3 of 6)

Step 4 - Find the point associated with the *y*-intercept.

We must set *x* equal to 0!

$h(0) = 0^2 - 2(0) + 1 = 1$. This is the *y*-intercept.

The coordinates of the point associated with the *y*-intercept are (0, 1).

Step 5 – Find the equation of the axis of symmetry. Use $x = -\frac{b}{2a}$.

The equation of the axis of symmetry is $x = \frac{-(-2)}{2(1)} = 1$.

Example 3: Graph a Quadratic Function in General Form (4 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (1, 0).
- The coordinates of the point associated with the *x*-intercept are (1, 0)
- The coordinates of the point associated with the *y*-intercept are (0, 1).
- The axis of symmetry is a vertical line passing through the vertex.

We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape.

Example 3: Graph a Quadratic Function in General Form (5 of 6)

Step 6 continued

We are going to pick $x = -2, -1, 2, 3, 4$, and 5 and find the corresponding y-values.

$h(-2) = (-2)^2 - 2(-2) + 1 = 9$

$h(-1) = (-1)^2 - 2(-1) + 1 = 4$

$h(2) = (2)^2 - 2(2) + 1 = 1$

$h(3) = (3)^2 - 2(3) + 1 = 4$

$h(4) = (4)^2 - 2(4) + 1 = 9$

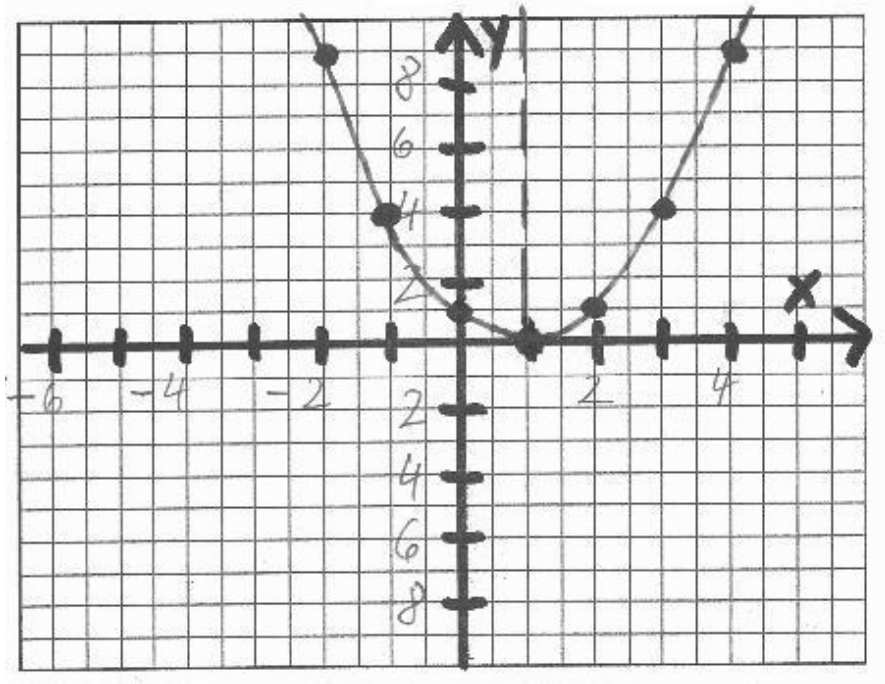
Example 3: Graph a Quadratic Function in General Form (6 of 6)

Step 6 continued

The coordinates of the additional points are

(– 2, 9), (– 1, 4), (2, 1), (3, 4), and (4, 9)

We will now add these points to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!



Example 4: Graph a Quadratic Function in Standard Form

Graph the quadratic function $p(x) = -(x - 1)^2 + 4$ by hand.



Step 1 - Determine how the parabola opens.

$a = -1$, $a < 0$; the parabola opens down.

Step 2 - Find the coordinates of the vertex.

The function is in standard form. Therefore, $h = 1$ and $k = 4$ and the parabola has its vertex at (1, 4).

Example 4: Graph a Quadratic Function in Standard Form

Step 3 - Find the point(s(associated with the x-intercept(s).

We must set $p(x) = y$ equal to 0!

Given $p(x) = -(x-1)^2 + 4$, let $f(x) = 0$ as follows:

$0 = -(x-1)^2 + 4$

Here we must solve a quadratic equation. We could do several things. We could multiply out and combine like terms, then try to factor and use the *Zero Product Principle*. We could also use the *Quadratic Formula*.

However, easiest is to use the *Square Root Property.* So, let's add $(x-1)^2$ to both sides of the equation to get

$(x-1)^2 = 4$

Example 4: Graph a Quadratic Function in Standard Form

(3 of 6)

Step 3 continued

We will now apply the *Square Root Property* to get

$x - 1 = \pm\sqrt{4} = \pm 2$

This means $x - 1 = 2$ and $x - 1 = -2$. It follows that $x = 3$ and $x = -1$. These are the x-intercepts.

The coordinates of the points associated with the x-intercepts are (3, 0) and (− 1, 0).

Example 4: Graph a Quadratic Function in Standard Form

Step 4 - Find the point associated with the *y*-intercept.

We must set *x* equal to 0!

$p(0) = -(0 - 1)^2 + 4 = -(-1)^2 + 4 = 3$. This is the *y*-intercept.

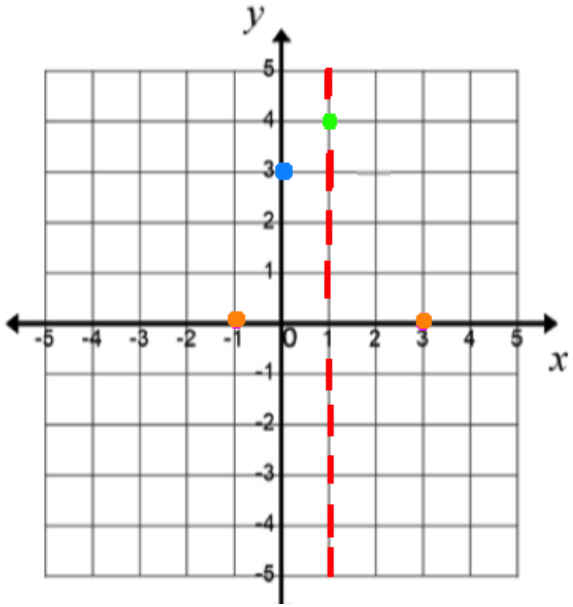
The coordinates of the point associated with the *y*-intercept are (0, 3).

Example 4: Graph a Quadratic Function in Standard Form

(5 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (1, 4).
- The coordinates of the points associated with the *x*-intercepts are (−1, 0) and (3, 0).
- The coordinates of the point associated with the *y*-intercept are (0, 3).



We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape. We are going to pick $x = 2$ and find its corresponding *y*-value.

$p(2) = -(2 - 1)^2 + 4 = 3$

The coordinates of the additional point are (2, 3).

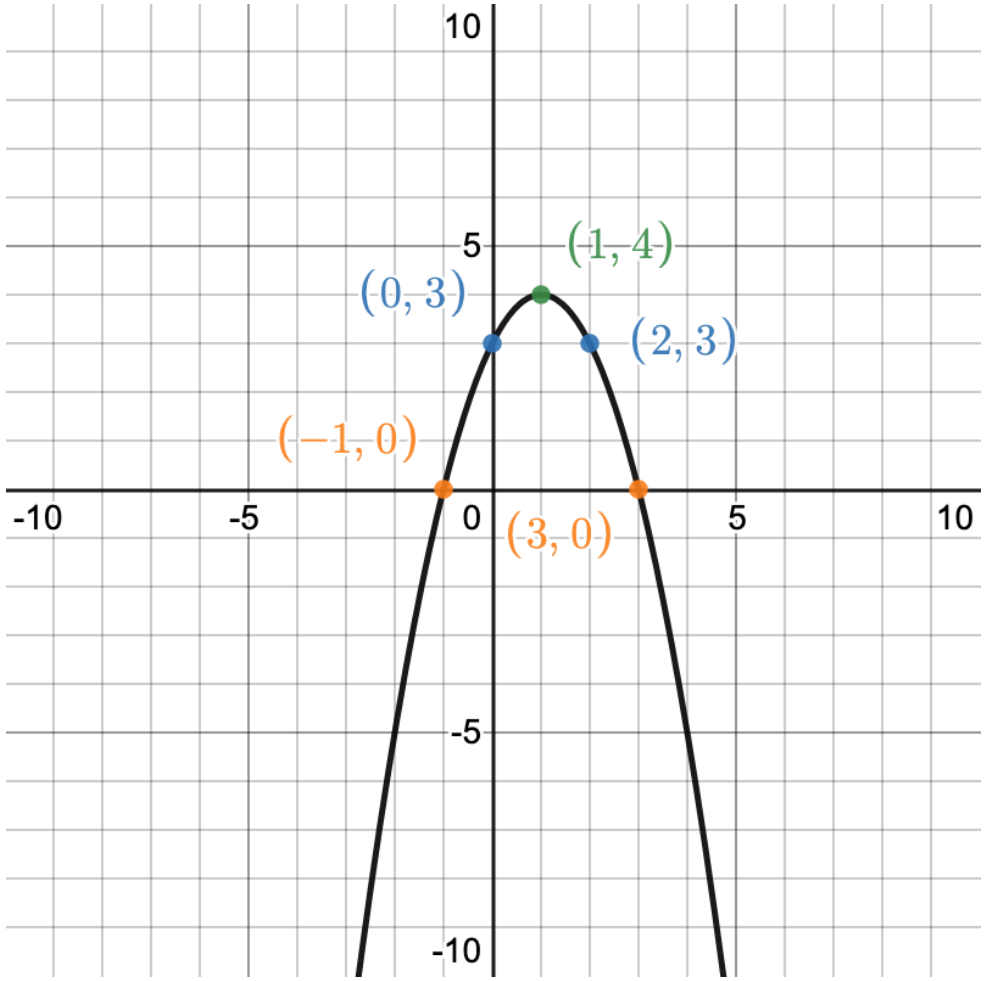
Example 4: Graph a Quadratic Function in Standard Form

(6 of 6)

Step 6 continued

We are now going to add the additional point (2, 3) to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!

Please note that you NEVER end a graph at the *x*-intercepts! Be sure to extend the graph well beyond them.



Example 5: Graph a Quadratic Function in Standard Form

Graph the quadratic function $g(x) = (x + 2)^2 - 1$ by hand.

Step 1 - Determine how the parabola opens.



$a = 1$, $a > 0$; the parabola opens up.

Step 2 - Find the coordinates of the vertex.

The function is NOT in standard form. Therefore, we first must change to standard from as follows:

$f(x) = (x - (-2))^2 + (-1)$

We now see that $h = -2$ and $k = -1$ and the parabola has its vertex at $(-2, -1)$.

Example 5: Graph a Quadratic Function in Standard Form

(2 of 6)

Step 3 - Find the point(s) associated with the *x*-intercept(s).

We must set $g(x) = y$ equal to 0!

Given $g(x) = (x + 2)^2 - 1$, let $f(x) = 0$ as follows:

$0 = (x + 2)^2 - 1$

Here we must solve a quadratic equation. We could do several things. We could multiply out and combine like terms, then try to factor and use the *Zero Product Principle*. We could also use the *Quadratic Formula*.

However, easiest is to use the *Square Root Property.* So, let's add 1 to both sides of the equation to get

$1 = (x + 2)^2$ or we can state $(x + 2)^2 = 1$.

Example 5: Graph a Quadratic Function in Standard Form

(3 of 6)

Step 3 continued

We will now apply the *Square Root Property* to get

$x + 2 = \pm\sqrt{1} = \pm 1$

This means $x + 2 = 1$ and $x + 2 = -1$. It follows that $x = -1$ and $x = -3$. These are the x-intercepts.

The coordinates of the points associated with the x-intercepts are $(-1, 0)$ and $(-3, 0)$.

Example 5: Graph a Quadratic Function in Standard Form

Step 4 - Find the point associated with the *y*-intercept.

We must set *x* equal to 0!

$g(0) = (0 + 2)^2 - 1 = (2)^2 - 1 = 3$. This is the *y*-intercept.

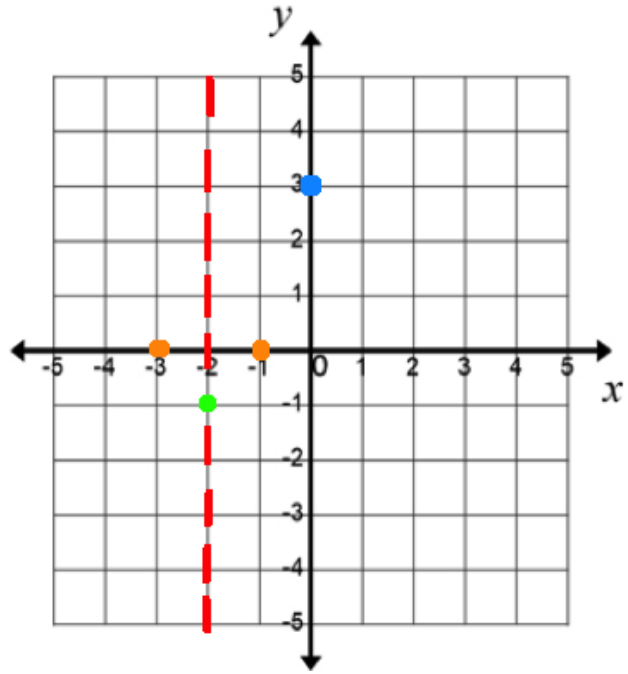
The coordinates of the point associated with the *y*-intercept are (0, 3).

Example 5: Graph a Quadratic Function in Standard Form

(5 of 6)

Step 6 - Graph the parabola.

- The coordinates of the vertex are (– 2, – 1).
- The coordinates associated with the *x*-intercepts are (–1, 0) and (– 3, 0).
- The coordinates of the point associated with the *y*-intercept are (0, 3).



We really do not have enough points to make an adequate graph of the function. Therefore, let's find a few more points particularly close to the vertex so that we can draw a nice bowl shape. We are going to pick $x = -4$ and find its corresponding *y*-value.

$g(2) = (-4 + 2)^2 - 1 = 3$

The coordinates of the additional point are (– 4 , 3).

Example 5: Graph a Quadratic Function in Standard Form

(6 of 6)

Step 6 continued

We are now going to add the additional point (– 4, 3) to the coordinate system and then connect ALL points keeping in mind the shape of a parabola!

Please note that you NEVER end a graph at points! Be sure to extend the graph well beyond them.

