Examples
Quadratic Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define quadratic equations in one variable.
2. Solve quadratic equations using the *Quadratic Formula*.
3. Solve quadratic equations using *factoring*.
4. Solve quadratic equations using the *Square Root Property*.

Example 1: Solve Quadratic Equations Using the Quadratic Formula (1 of 3)

Solve $x^2 + 9x + 16 = 0$ using the *quadratic formula*. Find only real solutions.

We will use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the given quadratic equation, $a = 1$, $b = 9$, and $c = 16$. We insert these values into the quadratic formula to get

$$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81 - 64}}{2}$$

$$x = \frac{-9 \pm \sqrt{17}}{2}$$

Example 1: Solve Quadratic Equations Using the Quadratic Formula (2 of 3)

The quadratic equation has two solutions, and they are $x = \frac{-9 + \sqrt{17}}{2}$ and $x = \frac{-9 - \sqrt{17}}{2}$.

These are irrational solutions because $\sqrt{17}$ is an irrational number.

Please note that in algebra, we usually leave answers in this form instead of changing them to decimal form.

Example 1: Solve Quadratic Equations Using the Quadratic Formula (3 of 3)

Just for calculator practice purposes, let's change $x = \frac{-9 + \sqrt{17}}{2}$ and $x = \frac{-9 - \sqrt{17}}{2}$ to decimal approximations.

We must tell the calculator that the numerator contains 2 terms. We will use the parentheses buttons to enclose the numerator. Once this is accomplished, we divide by 2.

We get the following:

$\frac{-9 - \sqrt{17}}{2} \cong -6.561552813$ … (infinitely many decimal places)

$\frac{-9 + \sqrt{17}}{2} \cong -2.438447187$ … (infinitely many decimal places)

Example 2: Solve Quadratic Equations Using the Quadratic Formula

Solve $x^2 - 2x = -2$ using the *quadratic formula*. Find only real solutions.

Before we can continue, we must first write the equation in the general form $ax^2 + bx + c = 0$. That is, $x^2 - 2x + 2 = 0$.

We can now see that $a = 1$, $b = -2$, and $c = 2$. We insert these values into the quadratic formula to get the following:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4-8}}{2}$$

and $x = \dfrac{2 \pm \sqrt{-4}}{2}$

We note that the solution contains the imaginary value $\sqrt{-4}$. Therefore, we can state that this quadratic equation has NO real solutions.

Example 3: Solve Quadratic Equations Using Factoring (1 of 2)

Solve $x^2 + 5x = -6$ by factoring. Find only real solutions.

Let's first change the equation to the general form $ax^2 + bx + c = 0$.

$$x^2 + 5x + 6 = 0$$

Next, we will write the quadratic expression as a product of factors.

Find all pairs of positive integers whose product is $c = 6$.

$6 = (1)(6)$ and $6 = (2)(3)$ and $6 = (-1)(-6)$ and $6 = (-2)(-3)$

Using the pairs above, find one whose sum is $b = 5$.

We notice that 2 and 3 have a sum of 5.

Example 3: Solve Quadratic Equations Using Factoring

Let's create the template $(x \quad)(x \quad)$ and use 2 and 3 as the second terms.

$$(x + 2)(x + 3) = 0$$

Now, we will solve this equation using the *Zero Product Principle*:

$$x + 2 = 0 \text{ or } x + 3 = 0$$

Then $x = -2$ or $x = -3$

See see that the quadratic equation has two integer solutions, and they are $x = -2$ and $x = -3$.

NOTE: We could have used the *quadratic formula* to get the same solutions.

Example 4: Solve Quadratic Equations Using Factoring (1 of 2)

Solve $x^2 - 4x = -4$ by factoring. Find only real solutions.

Let’s first change the equation to the general form $ax^2 + bx + c = 0$.

$$x^2 - 4x + 4 = 0$$

Next, we will write the quadratic expression as a product of factors.

Find all pairs of positive integers whose product is $c = 4$.

$4 = (1)(4)$ and $4 = (2)(2)$ and $4 = (-1)(-4)$ and $4 = (-2)(-2)$

Using the pairs above, find one whose sum is $b = -4$.

We notice that -2 and -2 have a sum of -4.

Example 4: Solve Quadratic Equations Using Factoring

Let's create the template $(x \quad)(x \quad)$ and use – 2 and – 2 as the second terms.

$$(x - 2)(x - 2) = 0$$

Now, we will solve this equation using the *Zero Product Principle*:

Since both factors are the same, we only need to set one equal to 0 as follows:

$x - 2 = 0$

and $x = 2$

The quadratic equation has one integer solution, and it is $x = 2$. **We could have used the *quadratic formula* to get the same solution.**

Example 5: Solve Quadratic Equations Using Factoring

Solve $2x^2 + 2x = 0$ by factoring. Find only real solutions.

In this equation, the c in $ax^2 + bx + c = 0$ missing. However, we note that both terms have a greatest common factor of $2x$. We will factor it out of each term as follows:

$$2x(x + 1) = 0$$

Now, we will solve this equation using the *Zero Product Principle*:

$2x = 0$ or $x + 1 = 0$

and $x = 0$ or $x = -1$

The quadratic equation has two integer solutions, and they are $x = 0$ and $x = -1$. **We could have used the *quadratic formula* to get the same solutions.**

Example 6: Solve Quadratic Equations Using the Square Root Property

Solve $x^2 = 16$ by the *Square Root Property*. Find only real solutions.

This equation is already in the form $u^2 = d$. Here, $u = x$ and $d = 16$.
Therefore, we can use the *Square Root Property* to state

$$x = \pm\sqrt{16} = \pm 4$$

The quadratic equation has two integer solutions, and they are – 4 and 4.

NOTE: We could have used the *quadratic formula* with $a = 1$, $b = 0$, and $c = -16$ to get the same solutions.

Example 7: Solve Quadratic Equations Using the Square Root Property

Solve $3x^2 - 21 = 0$ by the *Square Root Property*. Find only real solutions.

Before we can continue, we must first write the equation in the form $u^2 = d$.

$3x^2 - 21 = 0$
$3x^2 = 21$
$x^2 = 7$ Note, now $u = x$ and $d = 7$.

By the *Square Root Property,* we find that $x = \pm\sqrt{7}$.

This quadratic equation has two irrational solutions. They are $x = \sqrt{7}$ and $x = -\sqrt{7}$. We do not usually change them to decimal form!

NOTE: We could have used the *quadratic formula* with $a = 3$, $b = 0$, and $c = -21$ to get the same solutions.

Example 8: Solve Quadratic Equations Using the Square Root Property

Solve $2x^2 + 18 = 0$ by the *Square Root Property*. Find only real solutions.

Before we can continue, we must first write the equation in the form $u^2 = d$.

$2x^2 + 18 = 0$
$2x^2 = -18$
$x^2 = -9$ Note, now $u = x$ and $d = -9$.

By the *Square Root Property,* we find $x = \pm\sqrt{-9} = \pm 3i$.

Since the values for x are imaginary, we must state that this quadratic equation has "NO real solutions."

NOTE: We could have used the *quadratic formula* with $a = 2$, $b = 0$, and $c = 18$ to get the same solutions.

Example 9: Solve Quadratic Equations (1 of 2)

Solve $3x^2 + 27x = 0$. Find only real solutions.

Certainly, we could use the *Quadratic Formula* with $a = 3$, $b = 27$, and $c = 0$. Since the *Quadratic Formula* is sort of cumbersome to work with, our first thought should always be factoring or the *Square Root Property*.

While the given equation has only two terms, the *Square Root Property* will not easily work because neither term is a constant. So how about factoring?

We do notice that both terms have the factor of $3x$ in common which we can factor out as follows:

$3x(x + 9) = 0$

Now, we will solve this equation using the *Zero Product Principle.*

Example 9: Solve Quadratic Equations (2 of 2)

We set both factors equal to 0 as follows:

$3x = 0$ and $x + 9 = 0$

So that $x = 0$ and $x = -9$

We find that the quadratic equation has two integer solution, namely, $x = 0$ and $x = -9$.

NOTE: We could have used the *quadratic formula* to get the same solutions.