Examples
Linear and Constant Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define linear functions and graph them by hand.
2. Define constant functions and graph them by hand.
3. Define piecewise-defined functions.
4. Model real world problems with linear functions.

Example 1: Graph a Linear Function (1 of 4)

Graph the linear function $h(x) = 2x$ by hand.

Let's try the Intercept Method!

x-intercept:

Given $h(x) = 2x$, let $h(x) = y = 0$ and solve for x

$0 = 2x$

$0 = x$ (dividing both sides by 2!)

The *x*-intercept is 0 and the point associated with the *x*-intercept is (0, 0), which is the origin (the point at which the two coordinate axes intersect).

Example 1: Graph a Linear Function (2 of 4)

y-intercept:

Given $h(x) = 2x$, let $x = 0$ and solve for $h(x) = y$.

$h(0) = 2(0) = y$

The *y*-intercept is 0 and the point associated with the y-intercept is also (0, 0).

Example 1: Graph a Linear Function (3 of 4)

Most often the *Intercept Method* produces two different points. However, this does not always happen. In our case, the *Intercept Method* only produced one point, namely (0, 0).

Since we need at least two points to graph a line, we will use the *Point-by-Point Plotting Method* to find at least one more point. How about we find two points by letting x be equal to –2 and 2? This way we have a point above the origin and below the origin.

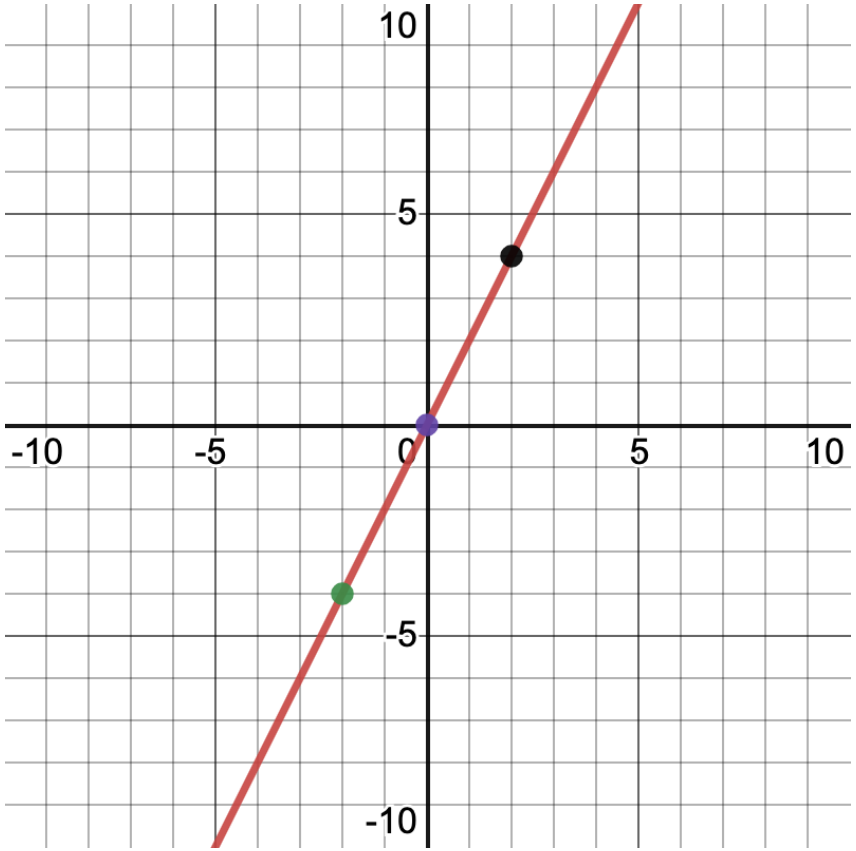
Given $h(x) = 2x$, we calculate $h(-2) = y = 2(-2) = -4$

and $h(2) = y = 2(2) = 4$

The coordinates of the additional points are (–2, – 4) and (2, 4).

Example 1: Graph a Linear Function (4 of 4)

Graph the function $h(x) = 2x$ by drawing a line through the points (0, 0), (–2, – 4), and (2, 4) found in the previous three slides.

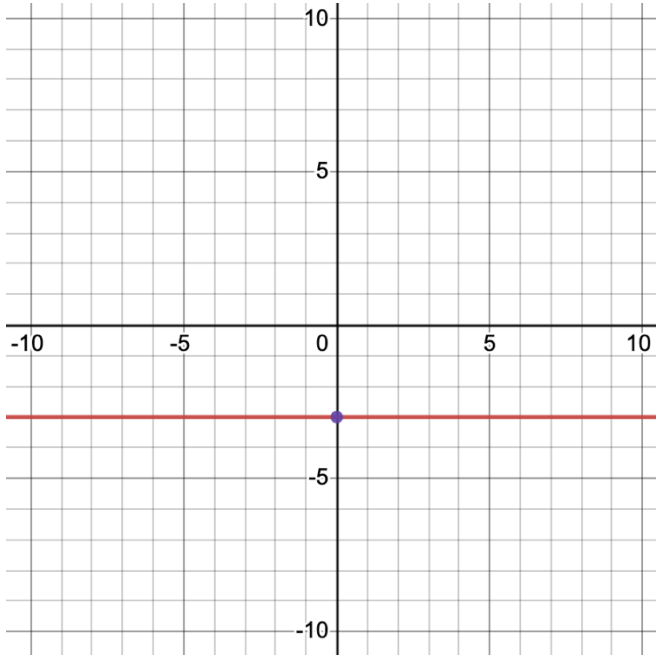


Example 2: Graph a Constant Function

Graph $k(x) = -3$ by hand.

It should be obvious that the function ***k*** is a *constant function*. As stated previously, the graph of the *constant function* is a horizontal line.

We know that the *y*-intercept is – 3, therefore the point associated with the *y*-intercept must be (0, – 3).



In the picture, we drew the point (0, – 3), which is the point associated with the *y*-intercept.

Then, we used our knowledge that the graph of the *constant function* is a horizontal line to draw a line through (0, – 3) parallel to the *x*-axis.

Example 3: Graph a Linear Function (1 of 3)

Graph the linear function $g(x)=\frac{3}{2}x-3$ by hand.

Let's try the Intercept Method!

x-intercept:

Given $g(x)=\frac{3}{2}x-3$, let $g(x) = y = 0$ and solve for x.

$$0=\frac{3}{2}x-3$$

$$3=\frac{3}{2}x$$

$$3\left(\frac{2}{3}\right)=x$$

$$2 = x$$

The *x*-intercept is 2 and the point associated with it is (2, 0).

Example 3: Graph a Linear Function (2 of 3)

y-intercept:

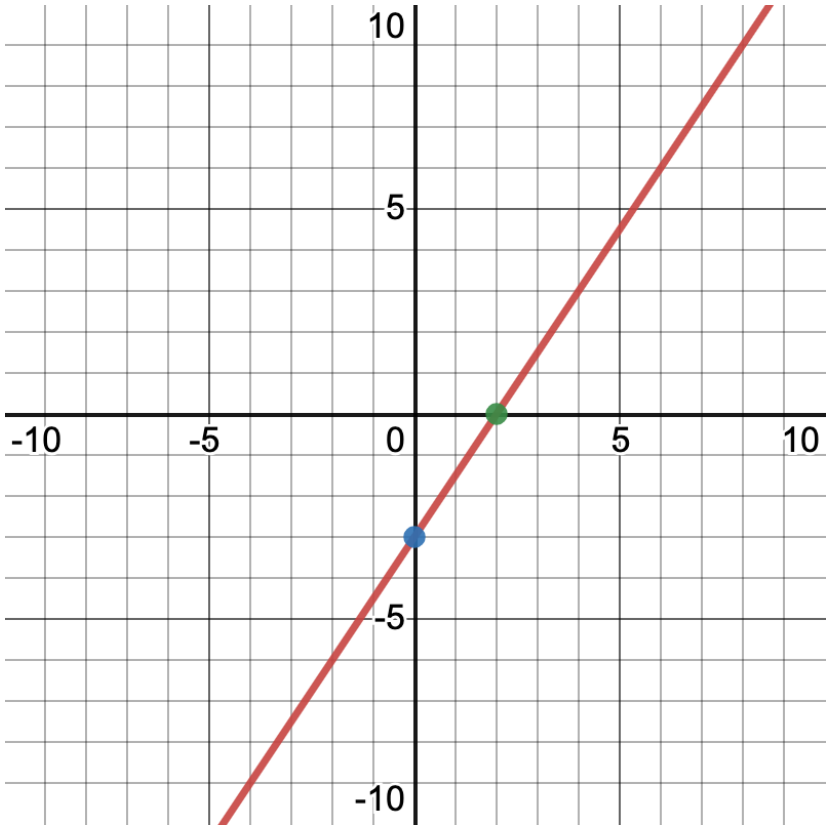
Given $g(x)=\frac{3}{2}x-3$, let $x = 0$ and solve for $g(x) = y$.

$$g(0)=\frac{3}{2}(0)-3=y$$

The y-intercept is − 3 and the point associated with it is (0, − 3).

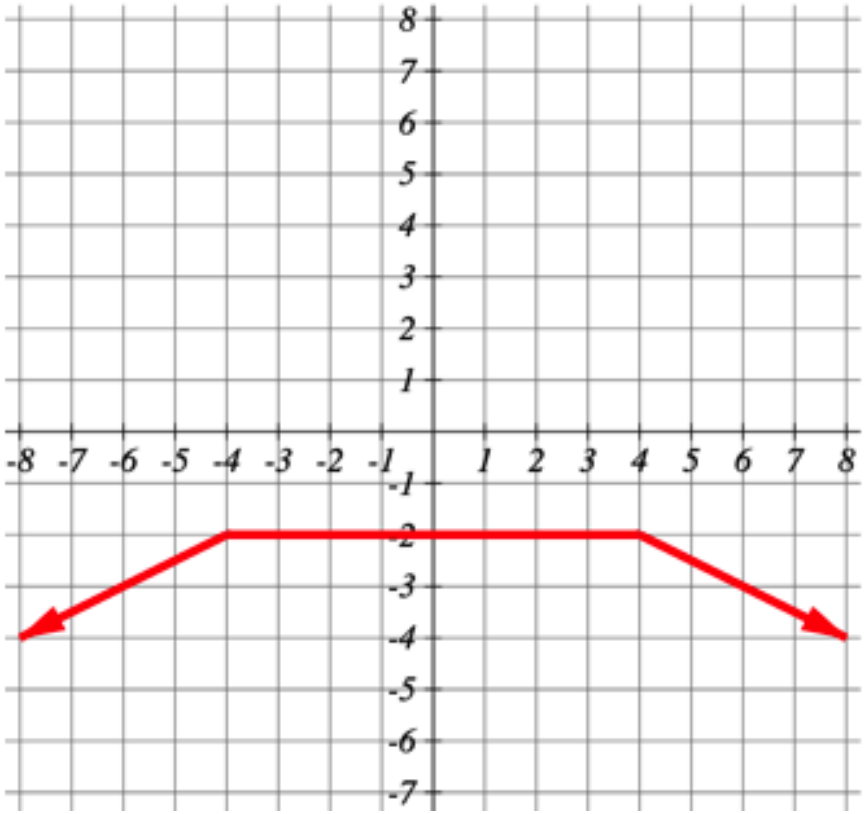
Example 3: Graph a Linear Function (3 of 3)

Graph the function $g(x) = \frac{3}{2}x - 3$ by drawing a line through the two points (2, 0) and (0, – 3) found in the previous slides.



Example 4: Piecewise Defined Functions

The graph below shows what is called a *piecewise-defined function*. Specifically, this one consists of three linear branches. In the middle, we have a horizontal line (constant function). On the left end it is joined to an increasing line and on the right end it is joined to a decreasing line which were produced by two linear functions.



Use the graph to find the following in *Interval Notation*. Use ***open intervals***, that is, use parentheses.

The function is increasing on which open interval along the *x*-axis: $(-\infty, -4)$

The function is decreasing on which open interval along the *x*-axis: $(4, \infty)$

The function is constant on which open interval along the *x*-axis: $(-4, 4)$

Example 5: Model Real World Problems with Linear Functions
(1 of 3)

In Seattle, Amtrack anticipates it will sell 300 tickets for their weekly trip to the Christmas Market in Leavenworth if the price of the ticket is $350 per person.
At $450 per person, Amtrack anticipates it will sell only 100 tickets for the same trip.

a. Find the demand function $d(x) = mx + b$, where x is the number of tickets sold, and $d(x) = y$ is the cost per ticket.

   We need m and b. The information given produces two ordered pairs, namely (300, 350) and (100, 450).

   Please note that in (a) we are told that the number of tickets sold represents the input variable.

Example 5: Model Real World Problems with Linear Functions
(2 of 3)

Given the two ordered pairs we can calculate *m* as follows:

$$m = \frac{450 - 350}{100 - 300} = -\frac{1}{2}$$

Next, we need *b*. We will use $d(x) = mx + b$, the point (300, 350), $m = -\frac{1}{2}$.

$$350 = -\frac{1}{2}(300) + b$$

$$350 = -150 + b$$

$$b = 500$$

Amtrack's demand function is $d(x) = -\frac{1}{2}x + 500$.

Example 5: Model Real World Problems with Linear Functions
(3 of 3)

b. Find the number of tickets sold if the ticket price per person is $415.

We will use the demand function we found in (a). That is, $d(x) = -\frac{1}{2}x + 500$ and replace $d(x)$ with 415.

$$415 = -\frac{1}{2}x + 500$$

$$-85 = -\frac{1}{2}x$$

$$x = 170$$

If the ticket price is $415, Amtrack will sell 170 tickets.