# Examples Linear Equations in One Variable

Based on power point presentations by Pearson Education, Inc.
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#### Learning Objectives

- 1. Memorize the *Basic Principles of Equations* consisting of four axioms.
- 2. Memorize what it means to solve an equation.
- 3. Solve linear equations containing integers and fractions.
- 4. Memorize and use the Cross-Multiplication Principle.

## Example 1: Solve a Linear Equation Containing Integers

Solve the linear equation x - 4 = 20.

Note that this equation is not in *general form*! We use the **Addition Axiom** and add 4 to both sides of the equal sign.

x - 4 + 4 = 20 + 4 (if you are comfortable, you do not have to show this step) and x = 24 which is the proposed solution.

Given x = 24, we now replace x in the original equation x - 4 = 20 with 24.

$$24 - 4 = 20$$

and 20 = 20

This is a true statement, therefore the proposed solution x = 24 is an actual solution.

## Example 2: Solve a Linear Equation Containing Integers

Solve the linear equation x + 5 = 11.

Note that his equation is not in *general form*! We use the **Subtraction Axiom** and subtract 5 from both sides of the equal sign.

x + 5 - 5 = 11 - 5 (if you are comfortable, you do not have to show this step) and x = 6 which is the proposed solution.

Given x = 6, we now replace x in the original equation x + 5 = 11 with 6.

$$6 + 5 = 11$$

and 
$$11 = 1$$

This is a true statement, therefore the proposed solution x = 6 is an actual solution.

## Example 3: Solve a Linear Equation Containing Integers (1 of 2)

Solve the linear equation 2x = 10.

Note that this equation is not in *general form*! We can either use the **Division Axiom** or the **Multiplication Axiom**. Let's use the Division Axiom first and then the Multiplication Axiom.

Let's divide both sides of the equal sign by 2 (which is the coefficient of the variable).

$$\frac{2x}{2} = \frac{10}{2}$$
 and  $x = 5$  which is the proposed solution.

Now let's use the Multiplication Axiom. That is, let's multiply both sides of the equal sign by  $\frac{1}{2}$  (which is the reciprocal of the coefficient of the variable).

$$\frac{1}{2}(2x) = \frac{1}{2}(10)$$
 and we also find that  $x = 5$ .

## Example 3: Solve a Linear Equation Containing Integers (2 of 2)

Given x = 5, we now replace x in the original equation 2x = 10 with 5.

$$2(5) = 10$$

and 10 = 10

This is a true statement, therefore the proposed solution x = 5 is an actual solution.

## Example 4: Solve a Linear Equation Containing Integers (1 of 2)

Solve the linear equation 2(x-4) - 5x = -5.

Note that this equation is not in *general form*! We will first simplify the expression on the left side of the equal sign using the *Distributive Property*.

$$2x-8-5x=-5$$
  
 $-3x-8=-5$  Combined like terms on the left.

Next, we collect variable terms on one side and constants on the other side.

$$-3x-8+8=-5+8$$
 Add 8 to both sides (Addition Axiom).  $-3x=3$  Combined like terms (simplified).

## Example 4: Solve a Linear Equation Containing Integers (2 of 2)

Finally, we "isolate" the variable. That is, let's divide both sides of the equal sign by – 3 (which is the coefficient of the variable).

$$\frac{-3x}{-3} = \frac{3}{3}$$
 and  $x = -1$  which is the proposed solution.

Given x = -1, we now replace x in the original equation 2(x - 4) - 5x = -5 with -1.

$$(-1-4)-5(-1)=-5$$
  
 $2(-5)+5=-5$   
 $-10+5=-5$   
 $-5=-5$ 

This is a true statement, therefore the proposed solution x = -1 is an actual solution.

## Example 5: Solve a Linear Equation Containing Integers (1 of 2)

Solve the linear equation 4(2x + 1) = 3(2x - 5) + 29.

Note that this equation is not in *general form*! We will first simplify the expressions on each side of the equal sign using the *Distributive Property*.

$$8x + 4 = 6x - 15 + 29$$

$$8x + 4 = 6x + 14$$

2x + 4 = 14

$$2x = 10$$

Combined like terms on the right.

Subtracted 6x from both sides (Subtraction Axiom) and combined like terms.

Subtracted 4 from both sides (Subtraction Axiom) and combined like terms.

## Example 5: Solve a Linear Equation Containing Integers (2 of 2)

Next, we "isolate" the variable. That is, let's divide both sides of the equal sign by 2 (which is the coefficient of the variable).

$$\frac{2x}{2} = \frac{10}{2}$$
 and  $x = 5$  which is the proposed solution

Given x = 5, we now replace x in the original equation 4(2x + 1) = 3(2x - 5) + 29 with 5.

$$4(2(5) + 1) = 3(2(5) - 5) + 29$$
  
 $4(11) = 3(5) + 29$   
 $44 = 15 + 29$   
 $44 = 44$ 

This is a true statement, therefore the proposed solution x = 5 is an actual solution.

## Example 6: Solve a Linear Equation Containing Integers

Solve the linear equation 2x + 6 = 2(x + 4).

Note that this equation is not in *general form*! We will first simplify the expression on the right side of the equal sign using the *Distributive Property*.

$$2x + 6 = 2x + 8$$
  
 $6 = 8$  Subtracted 2x from both sides (Subtraction Axiom).

Note, when we tried to solve for x, we ended up with 6 = 8, which is a FALSE statement.

In mathematics this means that the given equation has NO SOLUTION.

## Example 7: Solve a Linear Equation Containing Integers

Solve the linear equation 4x + 6 = 6(x + 1) - 2x.

Note that this equation is not in *general form*! We will first simplify the expression on the right side of the equal sign using the *Distributive Property*.

$$4x + 6 = 6x + 6 - 2x$$
  
 $4x + 6 = 4x + 6$  Combined like terms (simplified).  
 $6 = 6$  Subtracted 4x from both sides (Subtraction Axiom).

Note, when we tried to solve for x, we ended up with 6 = 6, which is a TRUE statement.

In mathematics this means that the given equation has INFINITELY MANY SOLUTIONS.

#### Example 8: Solve a Linear Equation Containing Fractions (1 of 2)

Solve the linear equation  $\frac{3}{4} - x = \frac{7}{8}$ .

Let's find a number divisible by all denominators in the equation. Please understand that we can always find such a number by calculating the product of all denominators. In our case, that would be 4(8) = 32.

However, you might notice that 8 is also a number divisible by 4 and 8. It certainly is smaller than 32 and therefore easier to work with. Subsequently, let's use 8 and multiply both sides of the equal sign by it.

$$8\left(\frac{3}{4}-x\right)=8\left(\frac{7}{8}\right)$$

$$8\left(\frac{3}{4}\right) - 8x = 8\left(\frac{7}{8}\right)$$
 Used the Distributive Property on the left of the equal sign.

#### Example 8: Solve a Linear Equation Containing Fractions (2 of 2)

After some canceling, we get 6 - 8x = 7 and -8x = 1.

All that's left to do is divide both sides of the equal sign by – 8 to get

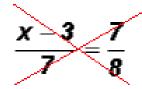
$$x=-\frac{1}{8}$$
.

If we wish, we could carry out a check to prove to ourselves that this is indeed the actual solution.

## Example 9: Solve a Linear Equation Containing Fractions (1 of 2)

Solve the linear equation  $\frac{x-3}{7} = \frac{7}{8}$  using Cross-Multiplication.

Let's make a cross over the equal sign as follows to help us remember *Cross-Multiplication*.



We can now state 8(x - 3) = 7(7).

Next, we must use the *Distributive Property* to get 8x - 24 = 49.

Using the **Addition Axiom**, we find 8x = 73.

## Example 9: Solve a Linear Equation Containing Fractions (2 of 2)

Next, we "isolate" the variable. That is, let's divide both sides of the equal sign by 8 (which is the coefficient of the variable).

We get 
$$x = \frac{73}{8}$$
.

If we wish, we could carry out a check to prove to ourselves that this is indeed the actual solution.

Now, should we change the improper fraction to a mixed number? Only if you are asked to do so. Otherwise, leave it alone.