Examples
Operations on Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Arithmetic operations on functions.
2. Composition of functions.

Example 1: Operations on Functions

Let $f(x) = -5x^3$ and $g(x) = 2x^3 - 1$. Find the functions $(f + g)(x)$ and $(f - g)(x)$, and simplify, if necessary.

$$(f + g)(x) = (-5x^3) + (2x^3 - 1)$$
$$= -5x^3 + 2x^3 - 1$$
$$= -3x^3 - 1$$

$$(f - g)(x) = (-5x^3) - (2x^3 - 1)$$
$$= -5x^3 - 2x^3 + 1$$
$$= -7x^3 + 1$$

Example 2: Operations on Functions

Let $f(x) = -5x^3$ and $g(x) = 2x^3 - 1$. Find the functions $(f\,g)(x)$ and $\left(\frac{f}{g}\right)(x)$ and simplify, if necessary.

$(f\,g)(x) = (-5x^3)(2x^3 - 1)$

$= -10x^3 + 5x^3$ (here we used the *Distributive Property* and the *power rule of exponents*!)

$\left(\frac{f}{g}\right)(x) = \frac{-5x^3}{2x^3 - 1}$ where the denominator $2x^3 - 1$ cannot be equal to 0!

Example 3: Operations on Functions

Let $h(x) = x^2 + 3$ and $k(x) = 2x - 1$. Find the functions $(h + k)(x)$ and $(h - k)(x)$, and simplify, if necessary.

$$(h + k)(x) = (x^2 + 3) + (2x - 1)$$
$$= x^2 + 3 + 2x - 1$$
$$= x^2 + 2x + 2$$

$$(h - k)(x) = (x^2 + 3) - (2x - 1)$$
$$= x^2 + 3 - 2x + 1$$
$$= x^2 - 2x + 4$$

Please note that the terms in a mathematical expression are ALWAYS displayed in descending order of their exponent!

Example 4: Operations on Functions

Let $h(x) = x^2 + 3$ and $k(x) = 2x - 1$. Find the functions $(hk)(x)$ and $\left(\frac{h}{k}\right)(x)$ and simplify, if necessary.

$(hk)(x) = (x^2 + 3)(2x - 1)$

Using FOIL, we get

$$\begin{matrix} & & & F & & O & & I & & L \\ (hk)(x) = (x^2 + 3)(2x - 1) & = & x^2(2x) & + & x^2(-1) & + & 3(2x) & + & 3(-1) \end{matrix}$$

$$= 2x^3 - x^2 + 6x - 3$$

$\left(\frac{h}{k}\right)(x) = \frac{x^2 + 3}{2x - 1}$ where the denominator $2x - 1$ cannot be equal to 0!

Example 5: Operations on Functions

Let $f(x) = -5x^3$ and $g(x) = 2x - 4$. Find $(f + g)(-2)$ and $\left(\frac{f}{g}\right)(-2)$, and simplify, if necessary.

Let's first find $f(-2)$ and $g(-2)$.

Then $f(-2) = -5(-2)^3 = 40$ and $g(-2) = 2(-2) - 4 = -8$.

Since, $(f + g)(-2) = f(-2) + g(-2)$, then $(f + g)(-2) = 40 + (-8) = 32$.

Since $\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)}$, then $\left(\frac{f}{g}\right)(-2) = \frac{40}{-8} = -5$.

Example 6: Operations on Functions (1 of 2)

Let $f(x) = -5x^3$ and $g(x) = 2x - 4$. Find $(f - g)\left(\frac{1}{3}\right)$ and simplify the answer, if necessary.

Let's first find $f\left(\frac{1}{3}\right)$ and $g\left(\frac{1}{3}\right)$.

Then $f\left(\frac{1}{3}\right) = -5\left(\frac{1}{3}\right)^3 = -5\left(\frac{1}{27}\right) = -\frac{5}{27}$ and $g\left(\frac{1}{3}\right) = 2\left(\frac{1}{3}\right) - 4 = \frac{2}{3} - 4$.

In $g\left(\frac{1}{3}\right)$, we are subtracting numbers with one being a fraction. Therefore, we must first find the common denominator, which is (preferably) the smallest number that both denominators divide into without a remainder.

With 4 being an integer, we can write it as $\frac{4}{1}$. The common denominator is 3 and $\frac{4}{1}$ becomes $\frac{12}{3}$.

Example 6: Operations on Functions (2 of 2)

Then $\frac{2}{3} - 4$ becomes $\frac{2}{3} - \frac{12}{3} = -\frac{10}{3}$.

Finally, since $(f-g)\left(\frac{1}{3}\right) = f\left(\frac{1}{3}\right) - g\left(\frac{1}{3}\right)$, we find that $(f-g)\left(\frac{1}{3}\right) = -\frac{5}{27} - \left(-\frac{10}{3}\right)$.

Again, we are subtracting numbers, but this time both are fractions. We must first find the common denominator.

The smallest number that both 27 and 3 divide into without a remainder is 27. Therefore, we can write $-\frac{10}{3}$ as $-\frac{90}{27}$.

Then $(f-g)\left(\frac{1}{3}\right) = -\frac{5}{27} - \left(-\frac{90}{27}\right) = -\frac{5}{27} + \frac{90}{27} = \frac{85}{27}$.

The answer is reduced to lowest terms! Therefore, there is nothing to simplify. Please note that in algebra we do not change this answer to a mixed number or a decimal number. Algebra "likes" fractional answer if at all possible and appropriate.

Example 7: Operations on Functions (1 of 2)

Let $f(x) = -5x^3$ and $g(x) = 2x - 4$. Find $(fg)\left(\frac{2}{5}\right)$ and simplify the answer, if necessary.

Let's first find $f\left(\frac{2}{5}\right)$ and $g\left(\frac{2}{5}\right)$.

Then $f\left(\frac{2}{5}\right) = -5\left(\frac{2}{5}\right)^3 = -5\left(\frac{8}{125}\right) = -\frac{40}{125} = -\frac{8}{25}$ and $g\left(\frac{2}{5}\right) = 2\left(\frac{2}{5}\right) - 4 = \frac{4}{5} - 4$.

In $g\left(\frac{2}{5}\right)$, we are subtracting numbers with one being a fraction. Therefore, we must first find the common denominator, which is (preferably) the smallest number that both denominators divide into without a remainder.

With 4 being an integer, we can write it as $\frac{4}{1}$. The common denominator is 5 and $\frac{4}{1}$ becomes $\frac{20}{5}$.

Example 7: Operations on Functions (2 of 2)

Then $\frac{4}{5} - 4$ becomes $\frac{4}{5} - \frac{20}{5} = -\frac{16}{5}$.

Finally, since $(fg)\left(\frac{2}{5}\right) = f\left(\frac{2}{5}\right) g\left(\frac{2}{5}\right)$, we find that $(fg)\left(\frac{2}{5}\right) = -\frac{8}{25}\left(-\frac{16}{5}\right)$.

In $(fg)\left(\frac{2}{5}\right)$, we are multiplying two fractions. According to the fraction multiplication rule, we multiply each numerator and each denominator.

Then $(fg)\left(\frac{2}{5}\right) = \frac{128}{125}$.

The answer is reduced to lowest terms! Therefore, there is nothing to simplify.

Example 8: Operations on Functions

Given $f(x) = x - 5$ and $g(x) = 2x - 3$, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and simplify, if necessary.

$$(f \circ g)(x) = f[g(x)] = f(2x - 3)$$

$$= 2x - 3 - 5$$

$$= 2x - 8$$

This is the function f where x was replaced by $|2x - 3|$.

$$(g \circ f)(x) = g[f(x)] = g(x - 5)$$

$$= 2(x - 5) - 3$$

$$= 2x - 10 - 3$$

$$= 2x - 13$$

This is the function g where x was replaced by $x - 5$.

Please note that the results of $f \circ g$ and $g \circ f$ are NOT the same!

Example 9: Operations on Functions

Given $f(x) = x + 4$ and $g(x) = x - 1$, find $\boldsymbol{(f \circ g)(x)}$ and $\boldsymbol{(g \circ f)(x)}$ and simplify, if necessary.

$$(f \circ g)(x) = f[g(x)] = f(x-1)$$
$$= (x-1) + 4$$
$$= x + 3$$

This is the function f where x was replaced by $x - 1$.

$$(g \circ f)(x) = g[f(x)] = g(x+4)$$
$$= (x+4) - 1$$
$$= x + 3$$

This is the function g where x was replaced by $x + 4$.

In this example, the results of $\boldsymbol{f \circ g}$ and $\boldsymbol{g \circ f}$ are the same.

Example 10: Compose Functions

Given $f(x) = \sqrt{x}$ and $g(x) = x - 3$, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and simplify, if necessary.

$$(f \circ g)(x) = f[g(x)] = f(x-3)$$

$$= \sqrt{x-3}$$ This is the function f where x was replaced by $x - 3$.

$$(g \circ f)(x) = g[f(x)] = g(\sqrt{x})$$

$$= \sqrt{x} - 3$$ This is the function g where x was replaced by $\sqrt{x}$.

In this example, the results of $f \circ g$ and $g \circ f$ are NOT the same.