# Concepts Uses of the Slope of a Line

Based on power point presentations by Pearson Education, Inc. Revised by Ingrid Stewart, Ph.D.

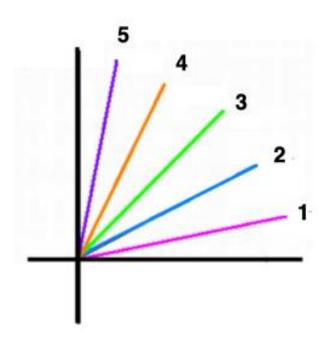
#### Learning Objectives

- 1. Define and find the slope of a line.
- Define the slope-intercept form of a linear equation in two variables.
- 3. Graph linear equations in slope-intercept form by hand in the rectangular coordinate system using the *Point-by-Point Plotting Method* and the *Intercept Method*.
- 4. Find the slope-intercept equation of a line.
- 5. Identify and use slopes of parallel and perpendicular lines.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

# 1. Definition of the Slope of a Line (1 of 6)

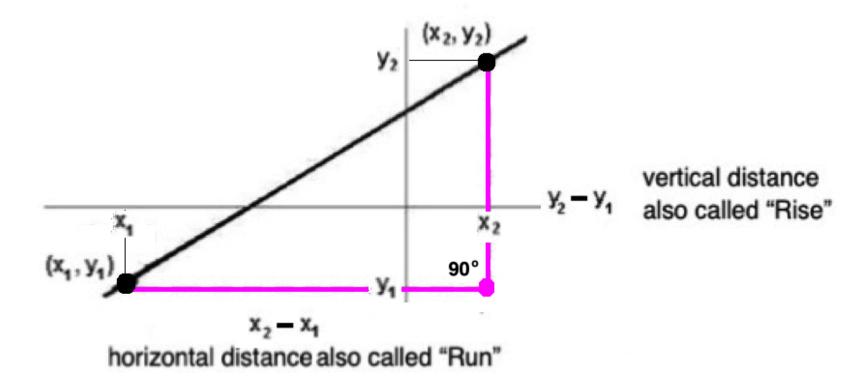
In the last lesson, we discussed the graphs of linear equations in two variables. We found out that their graphs are either increasing or decreasing lines. Let's look at a picture of several increasing lines.



We notice that each line has a different steepness. For example, the purple line #5 is much steeper than the pink line #1.

#### Definition of the Slope of a Line (2 of 6)

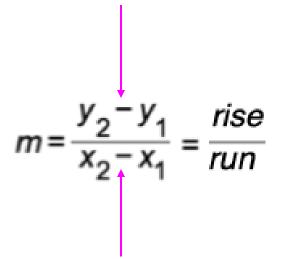
In mathematics, the steepness of a line, usually called the **slope**, is measured by an entirely man-made formula. Specifically, it was defined to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point on a line in a rectangular coordinate system to another point. Usually, these two points are defined by the ordered pairs  $(x_1, y_1)$  and  $(x_2, y_2)$ .



#### Definition of the Slope of a Line (3 of 6)

The slope of a line is indicated by the lower-case letter m. Why m? No one knows for sure. Some mathematicians claim the m comes from the French word "monter" which means "to climb".

The slope of the line through two distinct ordered pairs  $(x_1, y_1)$  and  $(x_2, y_2)$  is defined by the following formula:



Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

NOTE: The steeper the line, the larger is the *m*.

#### Definition of the Slope of a Line (4 of 6)

#### Example 1:

Find the slope of the line passing through the points determined by the ordered pairs (4, -2) and (-1, 5).

We will let (4, -2) equal  $(x_1, y_1)$  and (-1, 5) equal  $(x_2, y_2)$ . However, you can also let (-1, 5) equal  $(x_1, y_1)$  and (4, -2) equal  $(x_2, y_2)$ . In either case, you will get the same answer.

Let's say that (4, -2) equals  $(x_1, y_1)$  and (-1, 5) equals  $(x_2, y_2)$ . Be sure not to get confused!

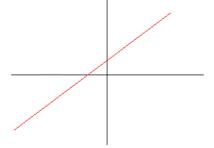
Then 
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{-1 - (+4)} = \frac{5 + 2}{-1 - 4} = \frac{7}{-5} = -\frac{7}{5}$$

# Definition of the Slope of a Line (5 of 6)

We will not state how certain slope properties affect the characteristics of a line.

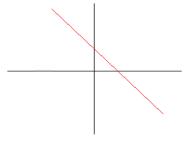
#### **POSITIVE SLOPE**

All increasing lines have a positive slope.



#### **NEGATIVE SLOPE**

ALL decreasing lines have a negative slope.



# Definition of the Slope of a Line (6 of 6)

#### SLOPE OF 0

All horizontal lines have a slope of 0. Slopes of 0 occur when the numerator in the slope formula is 0.

#### **UNDEFINED SLOPE**

All vertical lines have an undefined slope. Undefined slopes occur when the denominator in the slope formula is 0.

# 2. The Slope-Intercept Equation of a Line (1 of 4)

We have already discussed the **general form** of a linear equation in two variables. It is Ax + By + C = 0, where A, B, and C are numbers, but A and B cannot be 0.

We are now going to discuss the **slope-intercept form** of a linear equation in two variables. It is as follows:

y = mx + b where the number m is the slope of the line, and the number b is the y-intercept.

NOTE: The slope-intercept equation of a line is simply a mathematical "manipulation" of the general form with the introduction of the letter m.

#### The Slope-Intercept Equation of a Line (2 of 4)

#### Example 1:

Write the general form of the linear equation -18x - 2y + 11 = 0 in slope-intercept form.

We need to write the equation in the form y = mx + b. Note that the y-term is "isolated" on the left of the equal sign. The x-term and the constant are on the right side of the equal sign with the x-term in front of the constant.

Let's now do the following manipulations:

$$-2y + 11 = 18x$$
 (added 18x to both sides)

$$-2y = 18x - 11$$
 (subtracted 11 from both sides)

#### The Slope-Intercept Equation of a Line (3 of 4)

Example 1 continued with -2y = 18x - 11:

$$y = \frac{18x - 11}{-2}$$
 (divided both sides by -2)

As you can see, y is now isolated on one side. But we are not quite done!

We will now distribute -2 to every term in the numerator on the right side of the equal sign as follows:

$$y = \frac{18x}{-2} - \frac{11}{-2}$$
  
and  $y = -9x + \frac{11}{2}$ 

We find that -18x - 2y + 11 = 0 can be written as  $y = -9x + \frac{11}{2}$  in slope-intercept form.

#### The Slope-Intercept Equation of a Line (4 of 4)

Examples of linear equations written in *slope-intercept form*:

y = -18x + 11 (where m = -18 is the slope of the line and b = 11 is the y-intercept)

y = 2x (where m = 2 is the slope of the line and b = 0 is the y-intercept)

y = -x - 1 (this is still considered slope-intercept form) because the equation can be written as y = -x + (-1). Th

erefore, m = -1 is the slope of the line and b = -1 is the y-intercept.

Note: The *Slope-Intercept Form* requires a plus sign between the terms. Plus signs make no changes to the equations unlike minus signs.

# 3. The Graphs of Linear Equations in Slope-Intercept Form

We will graph linear equations in slope-intercept form the same way we graphed linear equation in the previous lesson. That is, we will use either the *Point-by-Point Plotting Method* or the *Intercept Method*.

NOTE: In a previous algebra courses, you may have learned a method of graphing called the "Slope-Intercept" Method. We will not use this method in our course.

# The Graphs of Linear Equations in Slope-Intercept Form (2 of 4)

#### Example 2:

Graph the linear equation y = -3x - 6 by hand. This linear equation is in slope-intercept form!

Since we are not told which graphing method to use, let's try to use the *Intercept Method*.

Find the ordered pair associated with the *y*-intercept.

Since the linear equation is in slope- intercept form, we know that b is the y-intercept, therefore, the y-intercept is -6

The ordered pair associated with the y-intercept is (0, -6).

# The Graphs of Linear Equations in Slope-Intercept Form (3 of 4)

Example 2 continued with y = -3x - 6:

Find the ordered pair associated with the *x*-intercept.

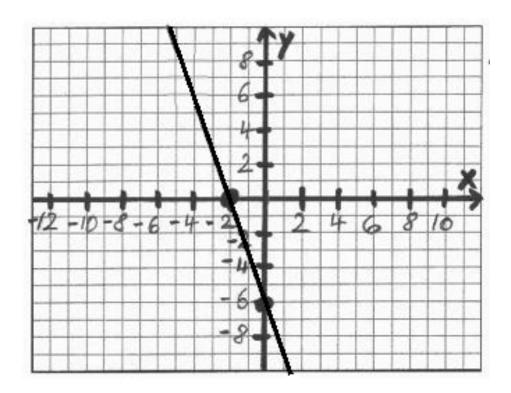
Let y = 0 and solve for x. 0 = -3x - 6 (this is a linear equation in one variable) 3x = -6x = -2

The x-intercept is -2, so the ordered pair associated with it is (-2, 0).

# The Graphs of Linear Equations in Slope-Intercept Form (4 of 4)

#### Example 2 continued:

Graph the linear equation by drawing a line through the points created by the ordered pairs associated with the y- and x-intercepts.



# 4. Find the Slope-Intercept Equation of a Line (1 of 6)

Often, we are asked to find the *slope-intercept equation* of a line. Sometimes we are given the slope m of the line and a point on the line created by the ordered pair  $(x_1, y_1)$ . Other times we are given two points on the line created by the ordered pairs  $(x_1, y_1)$  and  $(x_2, y_2)$ .

(1) Find the slope-intercept equation of a line given the slope m of the line and a point on the line determined by the ordered pair  $(x_1, y_1)$ 

**Step 1** – Place the given m and the point  $(x_1, y_1)$  into y = mx + b and find the value of b!

# Find the Slope-Intercept Equation of a Line (2 of 6)

#### Example 4:

Find the *slope-intercept equation* of a line whose graph has slope m = -4 and passes through the point determined by the ordered pair (-2, -3).

We will use the given m = -4 and (-2, -3) and place them into y = mx + b to find b as follows:

$$-3 = -4(-2) + b$$

$$-3 = 8 + b$$

$$-11 = b$$

# Find the Slope-Intercept Equation of a Line (3 of 6)

**Step 2** – Using m (given) and b (from Step 1) write the *slope-intercept* equation of the line.

Example 4 continued:

Given m = -4 and b = -11 and knowing y = mx + b, we can now write the *slope-intercept equation* of a line whose graph passes through the points created by the ordered pairs (-2, -3) and has slope -4.

That is, we get y = -4x + (-11) or preferably y = -4x - 11.

# Find the Slope-Intercept Equation of a Line (4 of 6)

(2) Find the *slope-intercept equation* of a line in two variables given two points on the line determined by the ordered pairs  $(x_1, y_1)$  and  $(x_2, y_2)$ .

**Step 1 –** Use  $(x_1, y_1)$  and  $(x_2, y_2)$  to find m.

Example 5:

Find the *slope-intercept equation* of a line whose graph passes through the points determined by the ordered pairs (-4, -6) and (-1, 3).

We need to find m. Let (-4, -6) equal  $(x_1, y_1)$  and (-1, 3) equal  $(x_2, y_2)$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-6)}{-1 - (-4)} = \frac{3 + 6}{-1 + 4} = \frac{9}{3} = 3$$

# Find the Slope-Intercept Equation of a Line (5 of 6)

**Step 2** – Place m (from Step 1) and ONE of the ordered pairs (given) into y = mx + b and find the value of b!

#### Example 5 continued:

We will use m = 3 and the given ordered pairs (-4, -6) and place them into y = mx + b to find the value of b as follows:

$$-6 = 3(-4) + b$$
 $-6 = -12 + b$ 
 $-6 + 12 = b$ 
 $6 = b$ 

# Find the Slope-Intercept Equation of a Line (6 of 6)

**Step 3** – Using *m* (from Step 1) and *b* (from Step 2) and write the *slope-intercept equation* of the line.

Example 5 continued:

Given m = 3 and b = 6 and knowing y = mx + b, we can now write the slope-intercept equation of the line whose graph passes through the points created by the ordered pairs (-4, -6) and (-1, 3).

That is, y = 3x + 6.

# 5. The Slopes of Parallel and Perpendicular Lines (1 of 6)

- 1. If two lines are parallel, then they have the same slope.
- 2. If two lines are perpendicular (intersect at 90° angles), then their slopes are negative reciprocals\*\*.

The **reciprocal**\*\* of a number is a new number in which the numerator and denominator of the given number is exchanged.

For example, the reciprocal of  $\frac{7}{11}$  is  $\frac{11}{7}$ .

Then the negative reciprocal is  $-\frac{11}{7}$ .

#### The Slopes of Parallel and Perpendicular Lines (2 of 6)

#### Example 6:

Are the lines created by y = 3x - 6 and y = 3x + 11 parallel or perpendicular or neither?

We notice that both lines have the same slope, namely m = 3. Therefore, the lines are parallel.

#### The Slopes of Parallel and Perpendicular Lines (3 of 6)

#### Example 7:

Are the lines created by y = -2x + 5 and  $y = \frac{1}{2}x - 4$  parallel or perpendicular or neither?

The lines are NOT parallel because they have different slopes.

Let's check if they are perpendicular.

We notice that the slope of the first line is -2 and the slope of the second line is  $\frac{1}{2}$ .

Now, the reciprocal of -2 which can be written as  $\frac{2}{1}$  is  $\frac{1}{2}$ . Then the negative reciprocal is  $-\left(-\frac{1}{2}\right) = \frac{1}{2}$ .

# The Slopes of Parallel and Perpendicular Lines (4 of 6)

Example 7 continued:

We see that the negative reciprocal of m = -2 is  $m = \frac{1}{2}$ .

Since the slope of the second line,  $y = \frac{1}{2}x - 4$ , is also  $\frac{1}{2}$ , we can say that the given lines are perpendicular.

#### The Slopes of Parallel and Perpendicular Lines (5 of 6)

#### Example 8:

Are the lines created by y = 3x + 1 and  $y = \frac{2}{3}x - 6$  parallel or perpendicular or neither?

The lines are certainly not parallel because they have different slopes. Let's check if they are perpendicular.

We notice that the slope of the first line is 3 and the slope of the second line is  $\frac{2}{3}$ .

Now, the reciprocal of 3 which can be written as  $\frac{3}{1}$  is  $\frac{1}{3}$ . Then the negative reciprocal is  $-\frac{1}{3}$ .

# The Slopes of Parallel and Perpendicular Lines (6 of 6)

Example 8 continued:

We see that the negative reciprocal of m = 3 is  $m = -\frac{1}{3}$ .

Since the slope of the second line,  $y = \frac{2}{3}x - 6$ , is NOT  $-\frac{1}{3}$ . we can say that the given lines are NEITHER parallel nor perpendicular.