Concepts
Quadratic Functions – Part 1

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define the *general form* of a quadratic function.
2. Recognize the characteristics of the graphs of quadratic functions.
3. Given a quadratic function in general form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
4. Graph quadratic functions in *general form* by hand.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. The General Form of a Quadratic Function (1 of 3)

We have already been exposed to quadratic equations in one variable, for example, $x^2 - 5x - 6 = 0$.

Now, we will discuss quadratic equations in two variables, for example, $y = x^2 - 5x - 6$, where x is the independent variable.

Quadratic equations in two variables are functions, therefore, we can replace the dependent variable with function notation, for example, $g(x) = x^2 - 5x - 6$.

The **general form** of the quadratic function in x is

$$\boldsymbol{f(x) = ax^2 + bx + c}, \text{ where } \boldsymbol{a, b,} \text{ and } \boldsymbol{c} \text{ are real numbers and } \boldsymbol{a} \neq 0$$

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

The General Form of a Quadratic Function

Examples of quadratic functions:

$g(x) = x^2 + 5x + 6$ ($a = 1$, $b = 5$, and $c = 6$)

$k(x) = 3x^2 + 21$ ($a = 3$, $b = 0$, and $c = 21$)

$p(x) = -4x^2 - 2x$

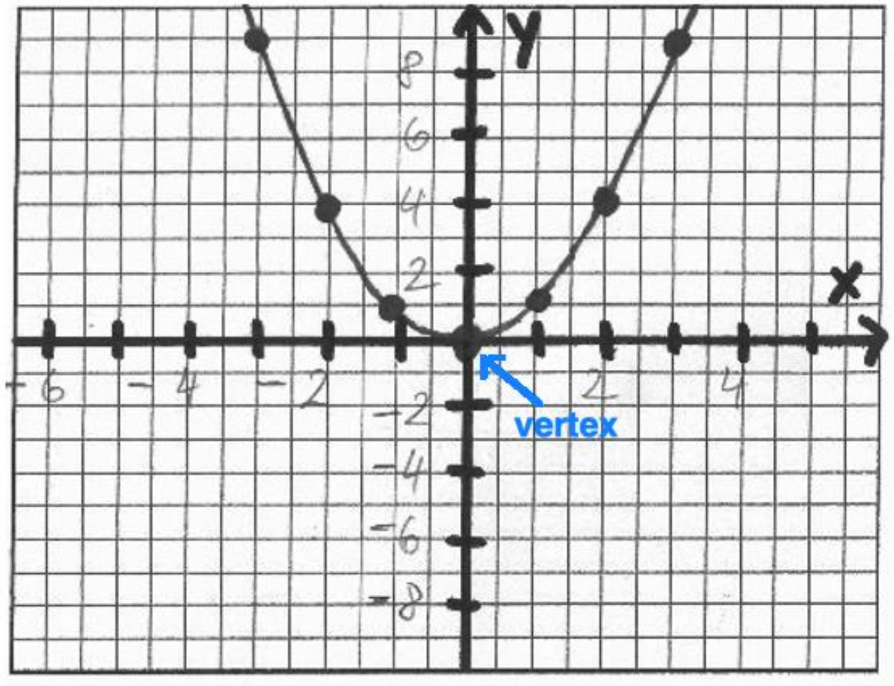
Please note that this function can be written as $p(x) = -4x^2 + (-2)x$. Now we see that $a = -4$, $b = -2$, and $c = 0$. We eliminate the double signs!

The General Form of a Quadratic Function (3 of3)

Examples of quadratic functions continued:

$f(x) = x^2$ ($a = 1$, $b = 0$, and $c = 0$)

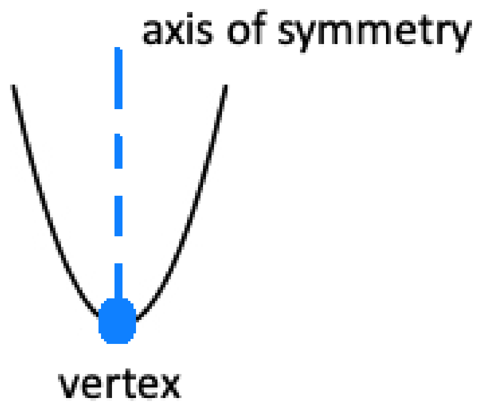
Special case of the quadratic function. It is often called the *Square Function*! We already discussed it in Lecture 8.



2. Characteristics of the Graphs of Quadratic Functions (1 of 3)

The graph of a quadratic function is called **parabola**. We can get graphs of parabolas open up and open down.

Parabolas Open Up occur when the coefficient $\boldsymbol{a}$ in $\boldsymbol{f(x) = ax^2 + bx + c}$ is greater than 0 ($a > 0$) or positive.

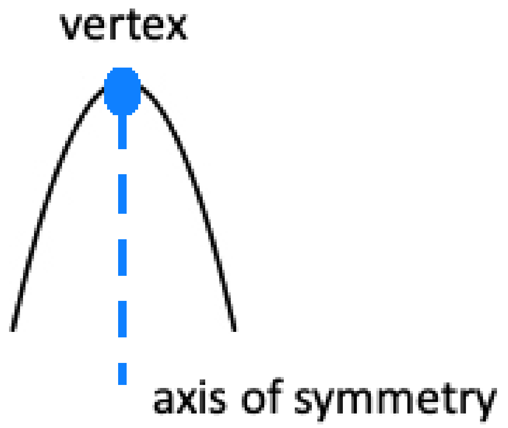


The vertex has the smallest *y*-coordinate of all points on the graph. Sometimes this *y*-coordinate is called a "minimum".

NOTE: The axis of symmetry is an invisible vertical line parallel to the *y*-axis that divides the parabola into two identical halves! Since it is invisible, we draw a dashed line!

Characteristics of the Graphs of Quadratic Functions (2 of 3)

Parabolas open down occur when the coefficient ***a*** in ***f(x) = ax² + bx + c*** is less than 0 ($a < 0$) or negative.



The vertex has the greatest *y*-coordinate of all points on the graph. Sometimes this *y*-coordinate is called a "maximum".

Please notice that the graphs of parabolas have a smooth curve around the vertex!

The General Form of a Quadratic Function

Example 1:

a. Is the graph of the quadratic function $g(x) = x^2 + 5x + 6$ a parabola open up or open down?

Since $a = 1$ which is greater than 0, the graph of the quadratic function is a parabola open up.

b. Is the graph of the quadratic function $p(x) = -4x^2 - 2x$ a parabola open up or open down?

Since $a = -4$ which is less than 0, the graph of the quadratic function is a parabola open down.

3. Coordinates of the Vertex and Equation of the Axis of Symmetry (1 of 3)

Given the *general form* $\boldsymbol{f(x) = ax^2 + bx + c}$,

- the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- the equation of the axis of symmetry (vertical line) is $x = -\frac{b}{2a}$.

NOTE: There is a proof in the learning materials showing that $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ are indeed the coordinates of the vertex.

Coordinates of the Vertex and Equation of the Axis of Symmetry (2 of 3)

Example 2:

Given the quadratic function $g(x) = -x^2 + 4x + 1$, find the coordinates of the vertex of its graph and the equation of the axis of symmetry. Write the location of the vertex point as an ordered pair.

The x-coordinate of the vertex using $-\frac{b}{2a}$:

In the given function, $a = -1$ and $b = 4$. Then the x-coordinate is $-\frac{4}{2(-1)} = 2$.

The y-coordinate of the vertex using $g\left(-\frac{b}{2a}\right)$:

$g(2) = -(2)^2 + 4(2) + 1$. **Please note that $-x^2 = -1(x^2)$.**

$= -4 + 8 + 1$

$= 5$

Coordinates of the Vertex and Equation of the Axis of Symmetry (3 of 3)

Example 2 continued:

Since $g(2) = 5$, we find that y-coordinate associated with $x = 2$ is 5.

Therefore, the coordinates of the vertex are (2, 5).

To find the <u>**equation**</u> of the axis of symmetry we use $x = -\frac{b}{2a}$.

We find that $x = -\frac{4}{2(-1)} = 2$.

4. Graph Quadratic Functions in General Form by Hand (1 of 2)

Graphing Strategy:

Step 1: Determine whether the parabola opens up or down.

Step 2: Find and plot the coordinates of the vertex of the parabola.

Step 3: If possible, find and plot the coordinates of point(s) associated with the x- intercept(s). There could be 0, 1, or 2.

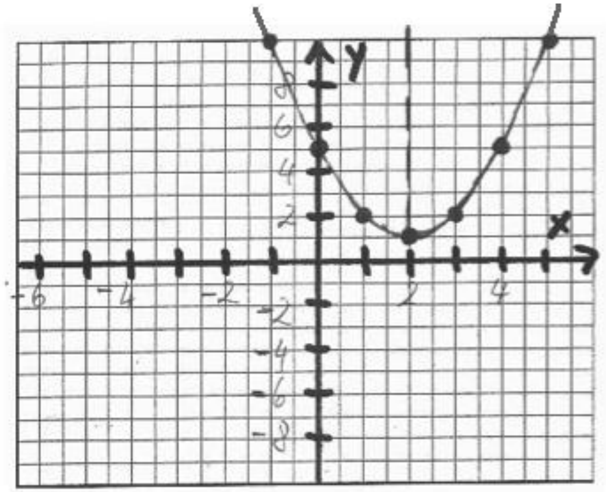
Step 4: Find and plot the coordinates of the point associated with the y-intercept. There is always exactly one y-intercept.

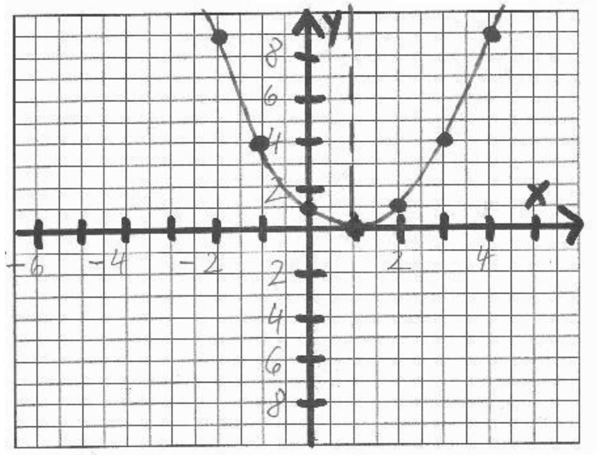
Step 5: Find and graph the axis of symmetry as a dashed line.

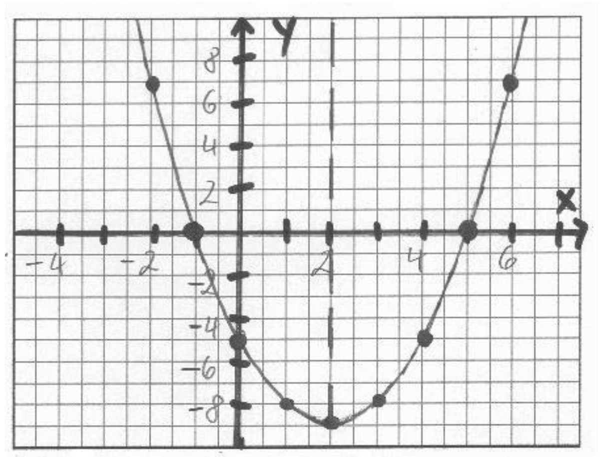
Step 6: Find and plot additional points, as necessary. Connect all points with a smooth curve that is shaped like a parabola open up or a parabola open down.

Graph Quadratic Functions in General Form by Hand (2 of 2)

The following examples show how there could be NO *x*-intercept, ONE *x*-intercept, or TWO *x*-intercepts.







For graphing examples, please refer to the "Examples" documents in the *MOM* Learning Materials folder pertaining to this lesson.