Concepts
Quadratic Functions

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define the general form of a quadratic function.
2. Define the standard form of a quadratic function.
3. Recognize the characteristics of the graphs of quadratic functions.
4. Given a quadratic function in general form and in standard form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
5. Graph quadratic functions by hand.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. The General Form of a Quadratic Function (1 of 3)

We have already been exposed to quadratic equations in one variable, for example, $x^2 - 5x - 6 = 0$.

Now, we will discuss quadratic equations in two variables, for example,

$y = x^2 - 5x - 6$, where x is the independent variable.

Quadratic equations in two variables are functions, therefore, we can replace the dependent variable with function notation, for example, $g(x) = x^2 - 5x - 6$.

The **general form** of the quadratic function in x is

$$\boldsymbol{f(x) = ax^2 + bx + c}, \text{ where } \boldsymbol{a}, \boldsymbol{b}, \text{ and } \boldsymbol{c} \text{ are real numbers and } \boldsymbol{a} \neq 0$$

Please note the plus signs!

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

The General Form of a Quadratic Function (2 of 3)

Examples of quadratic functions:

$g(x) = x^2 + 5x + 6$ ($a = 1$, $b = 5$, and $c = 6$)

$k(x) = 3x^2 + 21$ ($a = 3$, $b = 0$, and $c = 21$)

$p(x) = -4x^2 - 2x$

Please note that this function can be written as $p(x) = -4x^2 + (-2)x$. Now we see that $a = -4$, $b = -2$, and $c = 0$. We eliminate the double signs!

The General Form of a Quadratic Function (3 of3)

Examples of quadratic functions continued:

$f(x) = x^2$ ($a = 1$, $b = 0$, and $c = 0$)

Special case of the quadratic function. It is often called the *Square Function*! We already discussed it in a previous lecture.

2. The Standard Form of a Quadratic Function (1 of 3)

The **general form** of a quadratic function is defined to be $\boldsymbol{f(x) = ax^2 + bx + c}$. However, there is also a *standard form*.

Specifically, the **standard form** of the quadratic function in x is

$$\boldsymbol{f(x) = a(x - h)^2 + k}, \text{ where } a \neq 0$$

minus is part of the form!

Please note the plus sign!

The Standard Form of a Quadratic Function (2 of 2)

Examples of quadratic functions in standard form:

$g(x) = 4(x - 2)^2 + 1$ (standard form with $a = 4$, $h = +2$ and $k = +1$)

$t(x) = (x - 3)^2 - 5$

Please note that this function can be written as $(x - 3)^2 + (-5)$. Now we see that $h = +3$ and $k = -5$. We eliminate the double signs.

$p(x) = (x + 1)^2 + 4$

Please note that this function can be written as $(x - (-1))^2 + 4$. Now we see that $h = -1$ and $k = 4$. We eliminate the double signs.

The Standard Form of a Quadratic Function (3 of 2)

Example 1:

Change the quadratic function $g(x) = 4(x - 2)^2 + 1$ from *standard* to *general form*:

Given the standard form of a quadratic function, let's first eliminate the parentheses using FOIL and the we will combine like terms.

$g(x) = 4(x - 2)(x - 2) + 1$

$= 4(x^2 - 4x + 4) + 1$

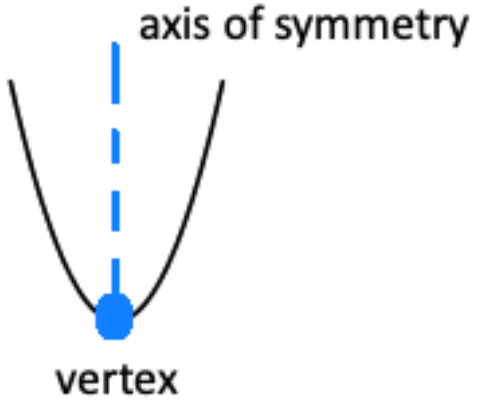
$= 4x^2 - 16x + 16 + 1$

$= 4x^2 - 16x + 17$ which is the general form.

3. Characteristics of the Graphs of Quadratic Functions (1 of 4)

The graph of a quadratic function is called **parabola**. We can get graphs of parabolas open up and open down.

Parabolas Open Up occur when the coefficient a in $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$ is greater than 0 ($a > 0$) or positive.

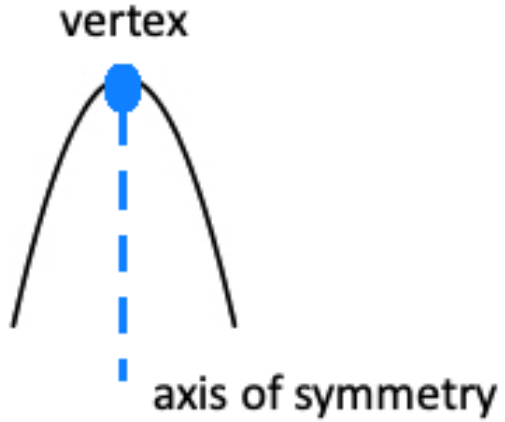


The vertex has the smallest *y*-coordinate of all points on the graph. Sometimes this *y*-coordinate is called a "minimum".

NOTE: The axis of symmetry is an invisible vertical line parallel to the *y*-axis that divides the parabola into two identical halves! Since it is invisible, we draw a dashed line!

Characteristics of the Graphs of Quadratic Functions (2 of 4)

Parabolas open down occur when the coefficient a in $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$ is less than 0 ($a < 0$) or negative.



The vertex has the greatest y-coordinate of all points on the graph. Sometimes this y-coordinate is called a "maximum".

Please notice that the graphs of parabolas have a smooth curve around the vertex!

Characteristics of the Graphs of Quadratic Functions (3 of 4)

Example 2:

a. Is the graph of the quadratic function $g(x) = x^2 + 5x + 6$ a parabola open up or open down?

Since $a = 1$ which is greater than 0, the graph of the quadratic function is a parabola open up. 

b. Is the graph of the quadratic function $p(x) = -4x^2 - 2x$ a parabola open up or open down?

Since $a = -4$ which is less than 0, the graph of the quadratic function is a parabola open down. 

Characteristics of the Graphs of Quadratic Functions (4 of 4)

Example 3:

a. Is the graph of the quadratic function $g(x) = 4(x - 2)^2 + 1$ a parabola open up or open down?

Since $a = 4$ which is greater than 0, the graph of the quadratic function is a parabola open up.

b. Is the graph of the quadratic function $h(x) = -(x + 3)^2 - 5$ a parabola open up or open down?

Since $a = -1$ which is less than 0, the graph of the quadratic function is a parabola open down.

4. Coordinates of the Vertex and Equation of the Axis of Symmetry (1 of 6)

Given the *general form* $\boldsymbol{f(x) = ax^2 + bx + c}$,

- the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- the equation of the axis of symmetry (vertical line) is $x = -\frac{b}{2a}$.

NOTE: There is a proof in the learning materials showing that $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ are indeed the coordinates of the vertex.

Coordinates of the Vertex and Equation of the Axis of Symmetry (2 of 6)

Example 4:

Given the quadratic function $g(x) = -x^2 + 4x + 1$, find the coordinates of the vertex of its graph and the equation of the axis of symmetry. Write the location of the vertex point as an ordered pair.

The x-coordinate of the vertex using $-\frac{b}{2a}$:

In the given function, $a = -1$ and $b = 4$. Then the x-coordinate is $-\frac{4}{2(-1)} = 2$.

The y-coordinate of the vertex using $g\left(-\frac{b}{2a}\right)$:

$g(2) = -(2)^2 + 4(2) + 1$. **Please note that $-x^2 = -1(x^2)$.**

$= -4 + 8 + 1$

$= 5$

Coordinates of the Vertex and Equation of the Axis of Symmetry (3 of 6)

Example 4 continued:

Since $g(2) = 5$, we find that y-coordinate associated with $x = 2$ is 5.

Therefore, the coordinates of the vertex are (2, 5).

To find the **equation** of the axis of symmetry we use $x = -\frac{b}{2a}$.

We find that $x = -\frac{4}{2(-1)} = 2$.

That is, the equation of the axis of symmetry is $x = 2$.

Coordinates of the Vertex and Equation of the Axis of Symmetry (4 of 6)

Given the standard form $f(x) = a(x - h)^2 + k$,

- the coordinates of the vertex are ***(h, k)***.
- the equation of the axis of symmetry is $x = h$.

There is a proof in the learning materials showing that *(h, k)* are indeed the coordinates of the vertex.

It also shows that *(h, k)* is equivalent to $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ **.**

Coordinates of the Vertex and Equation of the Axis of Symmetry (5 of 6)

Example 5:

Given the quadratic function $p(x) = (x - 2)^2 + 1$, find the coordinates of the vertex of its graph and the equation of the axis of symmetry. Write the location of the vertex point as an ordered pair.

This quadratic function is in *standard form*. Therefore, we can find h and k for the coordinates of the vertex.

We see that $h = 2$ and $k = 1$ and the coordinates of the vertex are (2, 1).

The equation of the axis of symmetry is $x = h$. Then $x = 2$.

Coordinates of the Vertex and Equation of the Axis of Symmetry (6 of 6)

Example 6:

Given the quadratic function $p(x) = (x + 3)^2 - 5$, find the coordinates of the vertex of its graph and the equation of the axis of symmetry. Write the location of the vertex point as an ordered pair.

This function is ALMOST in *standard form*. If we make a few sign changes, we can find h and k for the coordinates of the vertex.

Specifically, if we let $p(x) = (x - (-3))^2 + (-5)$ we have *standard form*.

We see that $h = -3$ and $k = -5$ and the coordinates of the vertex are $(-3, -5)$.

The equation of the axis of symmetry is $x = h$. Then $x = -3$.

5. Graph Quadratic Functions by Hand (1 of 2)

Graphing Strategy:

Step 1: Determine whether the parabola opens up or down.

Step 2: Find and plot the coordinates of the vertex of the parabola.

Step 3: If possible, find and plot the coordinates of point(s) associated with the *x*- intercept(s). There could be 0, 1, or 2. See next slide!

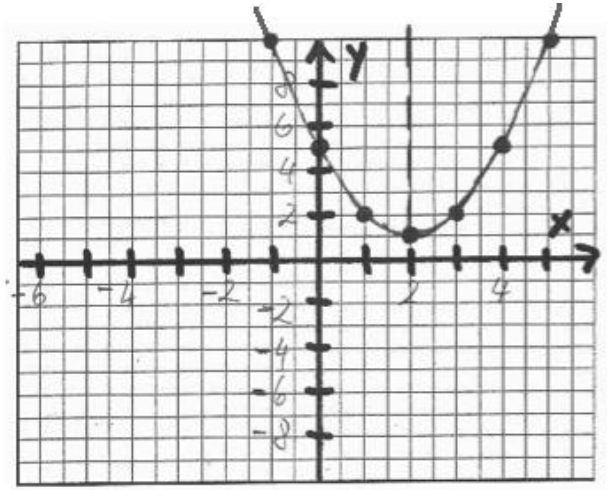
Step 4: Find and plot the coordinates of the point associated with the *y*-intercept. There is always exactly one *y*-intercept.

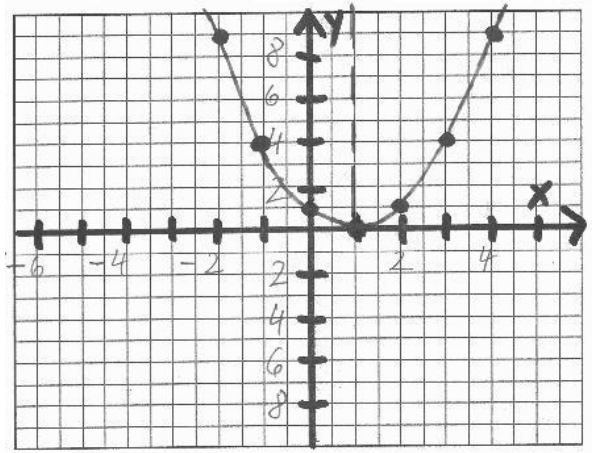
Step 5: Find and graph the axis of symmetry as a dashed line.

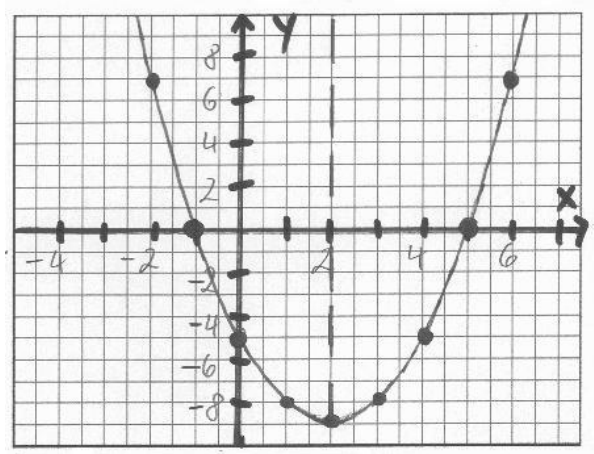
Step 6: Find and plot additional points, as necessary. Connect all points with a smooth curve that is shaped like a parabola open up or a parabola open down.

Graph Quadratic Functions in General Form by Hand (2 of 2)

The following examples show how there could be NO *x*-intercept, ONE *x*-intercept, or TWO *x*-intercepts.







For graphing examples, please refer to the ”Examples” documents in the *MOM* Learning Materials folder pertaining to this lesson.