Concepts
Quadratic Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define quadratic equations in one variable.
2. Solve quadratic equations using the *Quadratic Formula*.
3. Solve quadratic equations using *factoring*.
4. Solve quadratic equations using the *Square Root Property*.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Quadratic Equations in One Variable (1 of 4)

In a previous lesson, we discussed linear equations in one variable, say *x*, whose **general form** is $\boldsymbol{ax + b = 0}$.

Now, we are going to discuss quadratic equations in one variable, say *x*, whose **general form** is

$\boldsymbol{ax^2 + bx + c = 0}$, where a, b, and c are real numbers with $a \neq 0$.

Please note that quadratic equations in one variable do not necessarily have to appear in *general form*. Mathematics just likes to define them that way!

Quadratic Equations in One Variable

Examples of quadratic equations:

$x^2 + 5x + 6 = 0$ (general form with $a = 1$, $b = 5$, and $c = 6$)

$-2x^2 + 2x = 0$ (general form with $a = -2$, $b = 2$, and $c = 0$)

$3x^2 + 21 = 0$ (general form with $a = 3$, $b = 0$, and $c = 21$)

$7x^2 = 0$ (general form with $a = 7$, $b = 0$, and $c = 0$)

$x^2 - x - 3 = 0$

Please note that this equation can be written as $x^2 + (-1)x + (-3) = 0$. Now we see that $a = 1$, $b = -1$, and $c = -3$). We simply eliminated the double signs!

$2x^2 = x - 3$ (not in general form, but still a quadratic equation in one variable)

Quadratic Equations in One Variable (3 of 4)

There are several methods for solving quadratic equations. We commonly use the *quadratic formula*, *factoring*, or the *square root property*.

The *quadratic formula* can be used to solve all quadratic equations.

IF we notice that a quadratic equation is factorable, we do not have to use the quadratic formula to find the solutions. Instead, we can use *factoring*.

The *square root property* can usually be used to solve all quadratic equations. However, in this course we will only use it for certain types of quadratic equations.

Quadratic Equations in One Variable (4 of 4)

The solutions of quadratic equations can be real numbers or imaginary numbers. In this course, you are asked to only find **real** solutions. As soon as you get imaginary solutions, you will state that ”no real solutions exist.”

NOTE: A quadratic equation can have two (2) solutions or one (1) solution. The solutions can be integers, fractions, irrational numbers, or imaginary numbers.

2. Solve Quadratic Equations in One Variable Using the Quadratic Formula (1 of 3)

The *Quadratic Formula* states the following:

Given the general form of the quadratic equation $ax^2 + bx + c = 0$, its solutions for x are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The symbol $\pm$ indicates that there could potential be two solutions. One involving a negative square root and the other one a positive square root!

Note that the formula requires a, b, and c from the general form.

A discussion of where this formula comes from can be found in the learning materials.

Solve Quadratic Equations in One Variable Using the Quadratic Formula (2 of 4)

Strategy for solving quadratic equations using the *Quadratic Formula*:

Step 1: If necessary, write the quadratic equation in the **general form** $ax^2 + bx + c = 0$.

Example 1:

Solve $x^2 - 4x = -4$. Find only real solutions.

We will first write the equation in general form as $x^2 - 4x + 4 = 0$.

We can now see that $a = 1$, $b = -4$, and $c = 4$.

Solve Quadratic Equations in One Variable Using the Quadratic Formula (3 of 3)

Step 2: Insert the values of *a*, *b*, and *c* into the *Quadratic Formula* and solve for the variable.

Example 1 continued with $a = 1$, $b = -4$, and $c = 4$ in the *Quadratic Formula*:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)} = \frac{4 \pm \sqrt{0}}{2}$$

then $x = \frac{4+0}{2} = 2$ and $x = \frac{4-0}{2} = 2$

We find that the solution of $x^2 - 4x = -4$ consists of one real solution, that is, $x = 2$.

3. Solve Quadratic Equations in One Variable Using Factoring
(1 of 4)

Since a quadratic equation is of the general form $\boldsymbol{ax^2 + bx + c = 0}$, we can try to solve it using factoring instead of using the quadratic formula. However, we must then also use the *Zero Product Principle*.

Zero Product Principle states the following:

If A and B are two mathematical expressions and the product of A and B equals 0, then either A must be equal to 0 or B must be equal to 0 or both are equal to 0.

For example, if $(x + 1)(x – 6) = 0$, then either $(x + 1) = 0$ or $(x – 6) = 0$ or both equal 0.

Solve Quadratic Equations in One Variable Using Factoring (2 of 4)

Strategy for solving quadratic equations by factoring:

Step 1: If necessary, write the quadratic equation in the **general form** $ax^2 + bx + c = 0.$

Example 2:

Solve $x^2 – 5x = 6$. Find only real solutions.

We will first write the equation in general form as $x^2 – 5x – 6 = 0$.

Solve Quadratic Equations in One Variable Using Factoring

Step 2: Factor the quadratic expression.

Example 2 continued with $x^2 - 5x - 6 = 0$:

Let's find all pairs of positive integers whose product is $c = -6$.

$-6 = (1)(-6)$ and $-6 = (-2)(3)$ and $-6 = (-1)(6)$ and $-6 = (2)(-3)$

Using the pairs above, we find one whose sum is $b = -5$.

We notice that 1 and -6 have a sum of -5.

We now create the template $(x \quad)(x \quad)$ and use 1 and -6 as the second terms.

$(x + 1)(x - 6) = 0$

Solve Quadratic Equations in One Variable Using Factoring

Step 3: Set each factor equal to 0 (*Zero Product Principle*) and solve the resulting equations.

Example 2 continued with $(x + 1)(x - 6) = 0$:

We now set both factors equal to 0.

$x + 1 = 0$ and $x - 6 = 0$

Then $x = -1$ and $x = 6$

We find that $x^2 - 5x - 6 = 0$ has two real solutions, that is, $x = -1$ and $x = 6$.

Note that we could have solved the equation using the quadratic formula with $a = 1$, $b = -5$, and $c = -6$.

4. Solve Quadratic Equations in One Variable Using the Square Root Property (1 of 6)

Certain types of quadratic equations can sometimes easily be solved by the *Square Root Property* instead of using the *quadratic formula*.

The *Square Root Property* states the following:

Let ***u*** be a mathematical expression containing a variable and let ***d*** be a constant.

Then, the solutions of $u^2 = d$ are $u = \pm\sqrt{d}$, which means $u = \sqrt{d}$ and $u = -\sqrt{d}$.

Note that *u* must have a coefficient of 1.

Solve Quadratic Equations in One Variable Using the Square Root Property (2 of 6)

Examples of quadratic equations which are quickly solved using the *Square Root Property.*

$x^2 - 9 = 0$

$5x^2 - 15 = 0$

Please note they all only have a squared term and a constant.

Solve Quadratic Equations in One Variable Using the Square Root Property (3 of 6)

Strategy for solving quadratic equations using the *Square Root Property*:

Step 1: If necessary, write the quadratic equation in the form $u^2 = d$. Be sure that the coefficient of u^2 equals 1.

Example 3:

Solve $x^2 - 9 = 0$. Find only real solutions.

We will first write the equation as $x^2 = 9$.

Solve Quadratic Equations in One Variable Using the Square Root Property

Step 2: Use the *Square Root Property* and solve for *x.*

Example 3 continued:

Given $x^2 = 9$, we can use the *Square Root Property* to state the following:

$x = \pm\sqrt{9} = \pm 3$

The quadratic equation has two real solution, and they are $x = 3$ and $x = -3$.

Note that we could have used the quadratic formula with $a = 1$, $b = 0$, and $c = -9$.

Solve Quadratic Equations in One Variable Using the Square Root Property (5 of 6)

Example 4:

Solve $5x^2 - 15 = 0$ using the *Square Root Property*. Find only real solutions.

Before we can continue, we must first write the equation in the form $u^2 = d$, where u has a coefficient of 1.

$5x^2 - 15 = 0$

Let's first add 15 to both sides of the equal sign to get the following:

$5x^2 = 15$

But before we can use the *Square Root Property*, we must isolate the squared term.

Solve Quadratic Equations in One Variable Using the Square Root Property (6 of 6)

Example 4 continued with $5x^2 = 15$:

We will now divide both sides of the equal sign by 5, to get

$x^2 = 3$

Now, we can use the *Square Root Property* to state $x = \pm\sqrt{3}$.

We find that $5x^2 - 15 = 0$ has two real solutions, namely $x = \sqrt{3}$ and $x = -\sqrt{3}$. These solutions are irrational numbers. We do not change them to decimal form!

Note that we could have used the quadratic formula with $a = 5$, $b = 0$, and $c = -15$.