Concepts
Linear and Constant Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define linear functions and graph them by hand.
2. Define constant functions and graph them by hand.
3. Define piecewise-defined functions.
4. Model real world problems with linear functions.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Linear Functions and their Graphs (1 of 9)

When we discussed the *Vertical Line Test* in the previous lesson, we determined that linear equations in two variables are functions! Specifically, we can call them **linear functions**.

The linear function is defined, say in ***x,*** as ***f(x) = mx + b.***

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

The equation of the linear function is actually the slope-intercept equation of a line $y = mx + b$ where m is the slope and b the y-intercept.

Linear Functions and their Graphs (2 of 9)

Example 1:

Graph the linear function $\boldsymbol{g}(x) = -3x - 6$ by hand.

We will graph linear functions the same way we graphed linear equations in two variables. That is, we will use either the *Point-by-Point Plotting Method* or the *Intercept Method*.

Linear Functions and their Graphs (3 of 9)

Example 1 continued with $\boldsymbol{g}(x) = -3x - 6$:

Since we are not told which graphing method to use, let's use the *Intercept Method*.

Find the ordered pair associated with the *y*-intercept.

Since the linear function is in slope- intercept form, we know that *b* is the *y*-intercept, therefore, the *y*-intercept is – 6.

The ordered pair associated with the *y*-intercept is (0, – 6).

Linear Functions and their Graphs (4 of 9)

Example 1 continued with $\boldsymbol{g}(x) = -3x - 6$:

Find the ordered pair associated with the x-intercept.

Let $\boldsymbol{g}(x) = y = 0$ and solve for x.

$0 = -3x - 6$ (this is a linear equation in one variable)

$3x = -6$

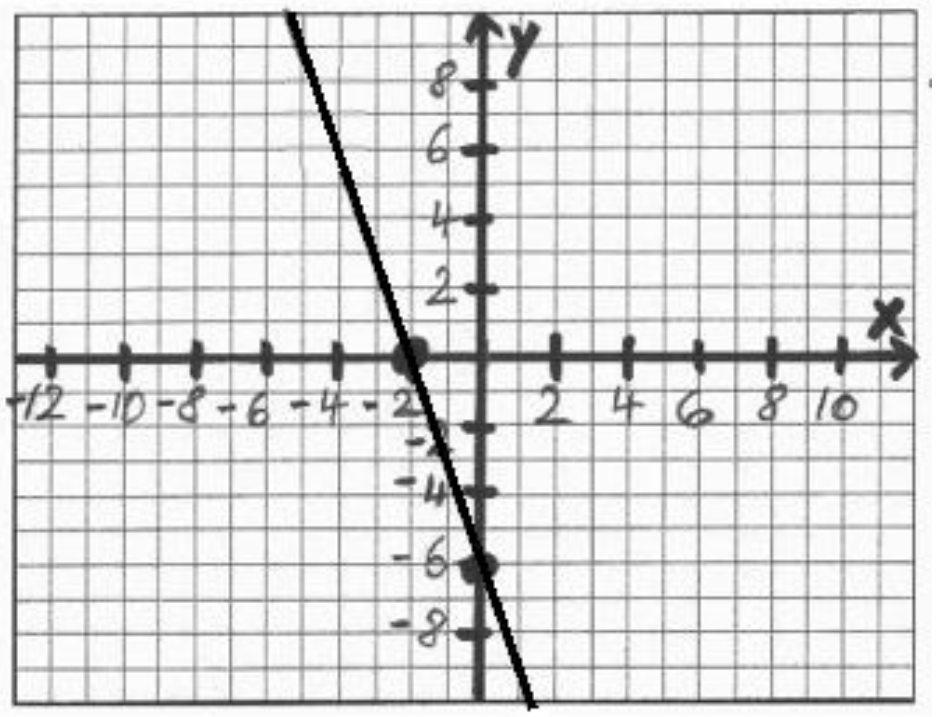
$x = -2$

The x-intercept is -2, so the ordered pair associated with it is $(-2, 0)$.

Linear Functions and their Graphs (5 of 9)

Example 1 continued:

Graph the linear function $\boldsymbol{g}(x) = -3x - 6$ by drawing a line through the points created by the ordered pairs associated with the y- and x-intercepts $(-2, 0)$ and $(0, -6)$.



Linear Functions and their Graphs (6 of 9)

Example 2:

Graph the linear function $\boldsymbol{h}(x) = x$ by hand.

NOTE: This particular linear function is also called **Identity Function**!

Since we are not told which graphing method to use, let's use the *Intercept Method*.

Find the ordered pair associated with the *y*-intercept.

Since the linear function is in slope- intercept form, we know that *b* is the *y*-intercept, therefore, the *y*-intercept is 0.

The ordered pair associated with the *y*-intercept is (0, 0).

Linear Functions and their Graphs (7 of 9)

Example 2 continued with $\boldsymbol{h}(x) = x$:

Find the ordered pair associated with the x-intercept.

Let $\boldsymbol{h}(x) = y = 0$ and solve for x.

$0 = x$

The x-intercept is 0, so the ordered pair associated with it is (0, 0).

Linear Functions and their Graphs (8 of 9)

Example 2 continued with $\boldsymbol{h}(x) = x$:

When the *Intercept Method* only produces one ordered pair, we use the *Point-by-Point Plotting Method* to find one or more other ordered pairs.

We want to pick values for x that are far enough away from 0 to create an accurate graph. How about we let x equal – 4 and 4?

Then, given the function $\boldsymbol{h}(x) = x$, we get the following:

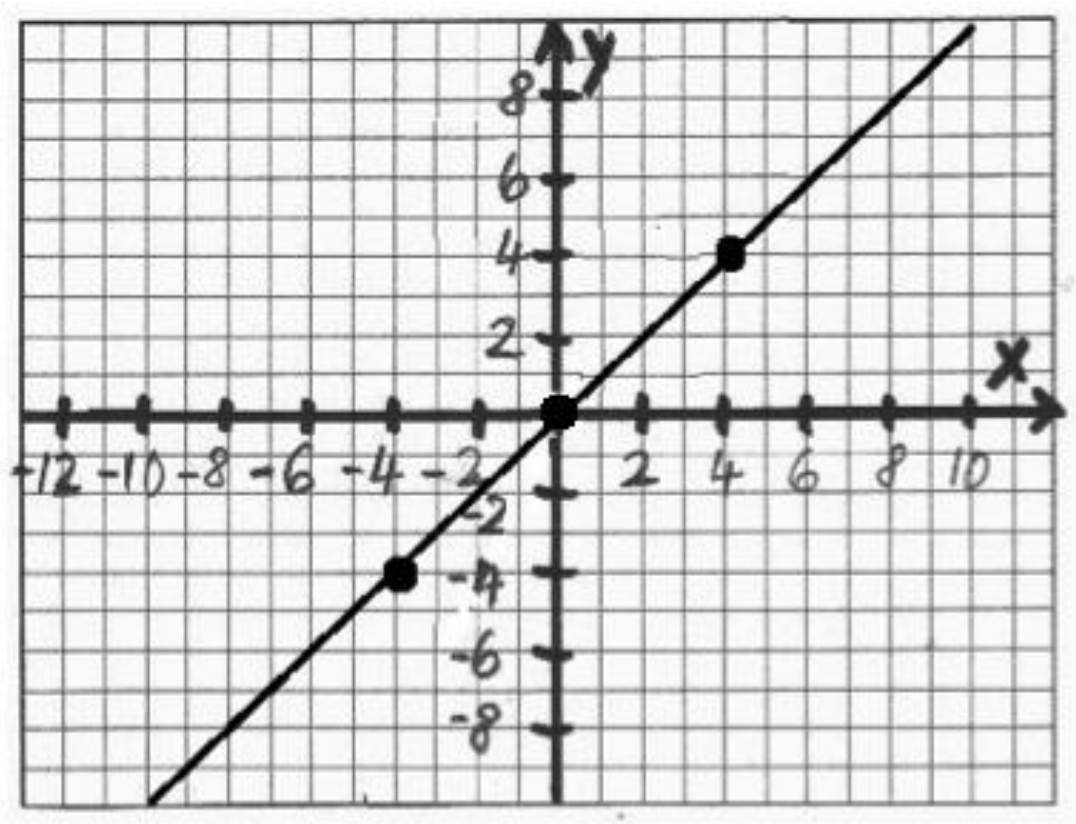
$\boldsymbol{h}(-4) = -4$ and $\boldsymbol{h}(4) = 4$

Therefore, the coordinates of the additional ordered pairs are (– 4, – 4) and (4, 4).

Linear Functions and their Graphs (9 of 9)

Example 2 continued:

Graph the linear function $\boldsymbol{h}(x) = x$ by drawing a line through the points created by the ordered pairs $(-4, -4)$, $(0, 0)$, and $(4, 4)$.



Observe that in an *Identity Function* *all* *y*-coordinates are equal to their respective *x*-coordinates.

2. Constant Functions and their Graphs (1 of 2)

The *Constant Function* is defined as ***f(x) = b***, where ***b*** is any real number.

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

The definition of a constant function is nothing but the equation $y = b$ whose graph is a horizontal line and b is the y-intercept.

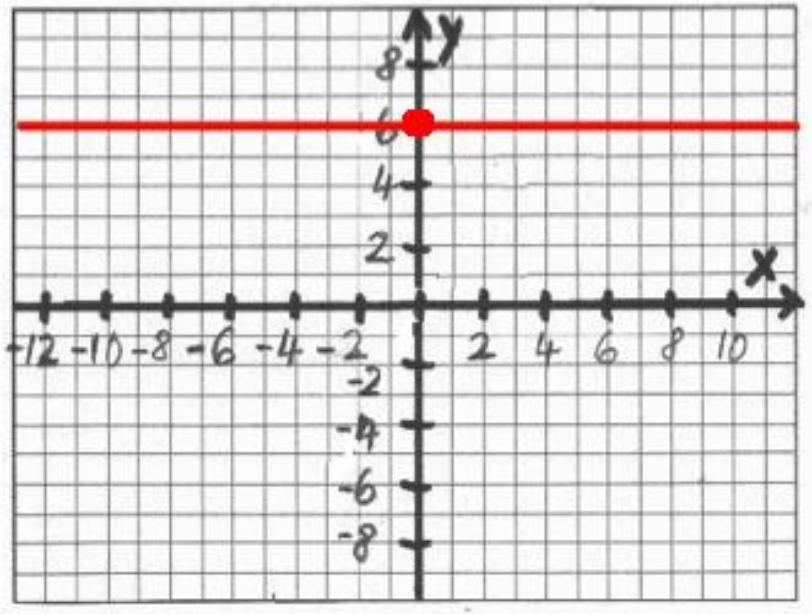
Please note that vertical lines are NOT functions. Do the *Vertical Line Test* to convince yourself!!!

Constant Functions and their Graphs (2 of 2)

Example 3:

Graph the function $\boldsymbol{g}(x) = 6$ by hand.

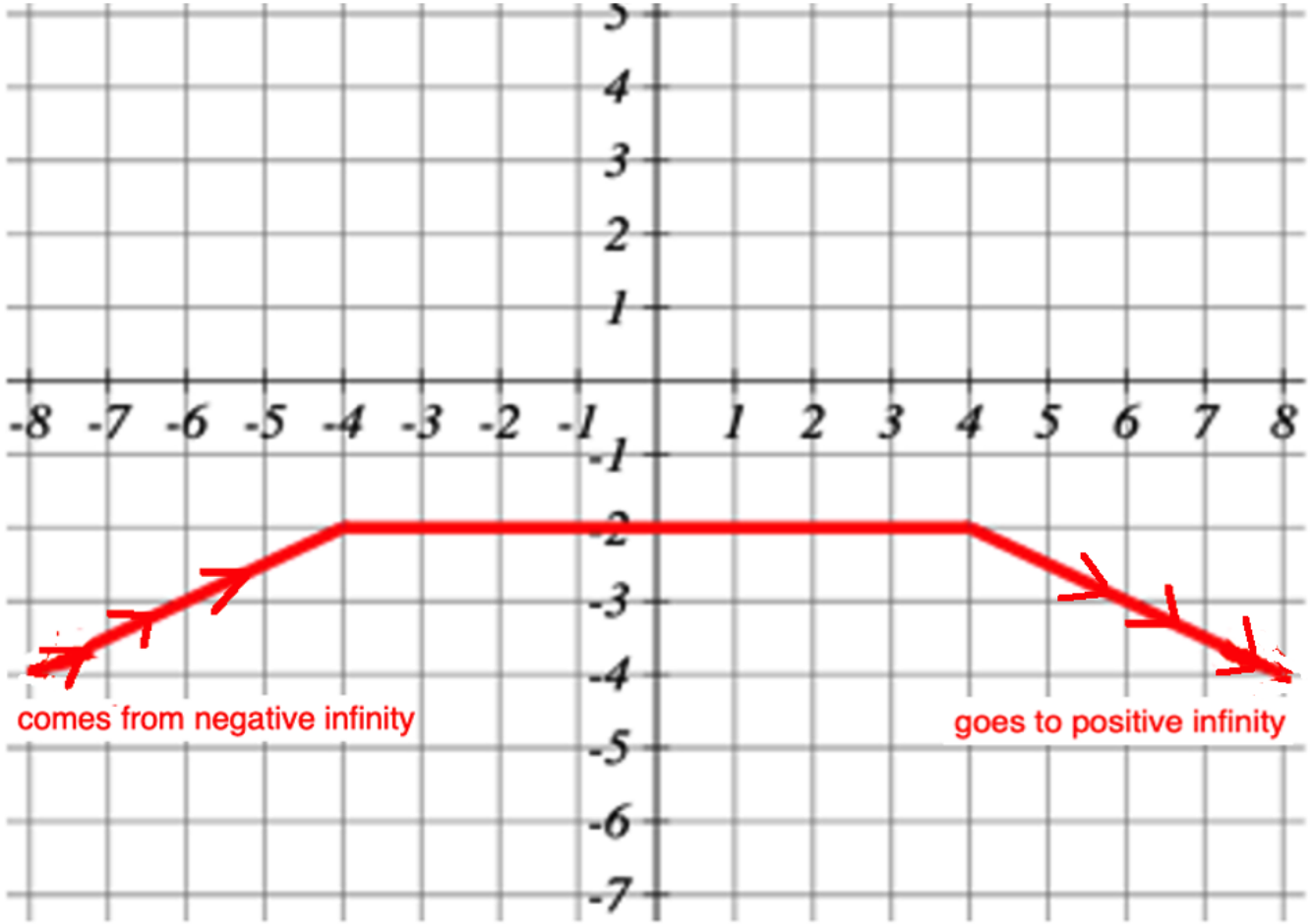
From the function we know that we are dealing with a *Constant Function* whose graph is a horizontal line parallel to the *x*-axis. We note that $b = 6$ which is the *y*-intercept. Therefore, the ordered pair associated with this intercept is (0, 6). Let's plot this ordered pair and then simply draw a horizontal line through it that is parallel to the *x*-axis.



You MUST memorize that a *Constant Function* is $\boldsymbol{f(x) = b}$ where $\boldsymbol{b}$ can be any real number.

3. Piecewise-Defined Functions (1 of 2)

Piecewise-defined functions are functions whose graphs are created from several ”pieces” of different functions. The graph below shows pieces of linear and constant functions.

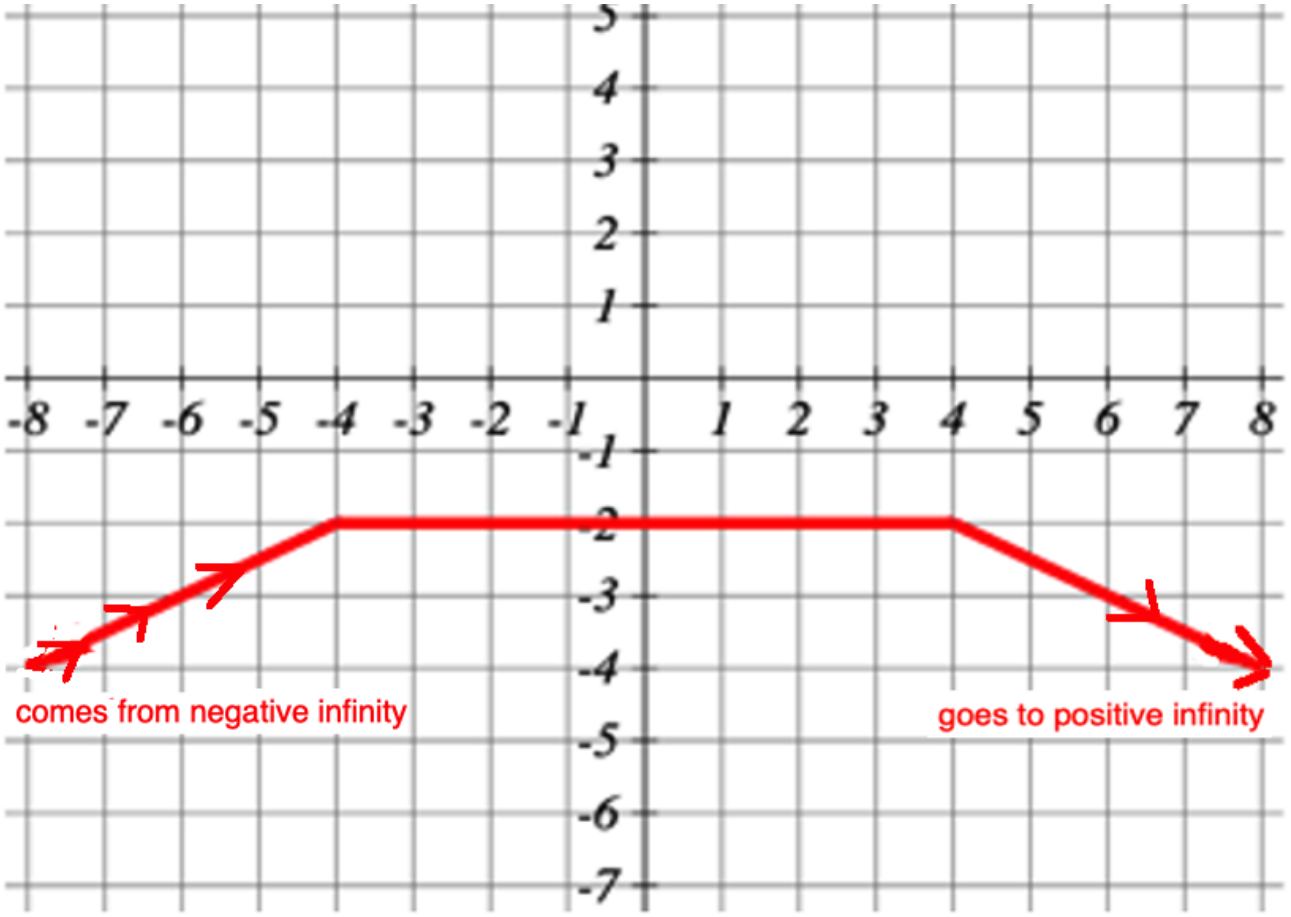


In the middle, we have a horizontal line (constant function).

The left end of the horizontal line is joined to an increasing line that comes from negative infinity.

The right end of the horizontal line is joined to a decreasing line that heads toward positive infinity.

Piecewise-Defined Functions (2 of 2)



Use the graph to find the following:

The function is increasing on which **open interval** along the *x*-axis?

$(-\infty, -4)$

The function is constant on which **open interval** along the *x*-axis?

$(-4, 4)$

The function is decreasing on which **open interval** along the *x*-axis?

$(4, \infty)$

4. Model Real World Problems with Linear Functions (1 of 6)

Now we will model real world problems using linear functions. When we do so, we will use the linear function definition $\boldsymbol{f(x) = mx + b}$.

Following is a strategy which allows us to write a linear model.

1. Read the problem carefully.
2. If necessary, assign a letter to the unknown quantity (what you are asked to find, e.g., cost, profit, revenue). It can be "*x*" or any other letter.
3. Find the slope *m* and the *y*-intercept *b*.
4. Write a linear model.

Model Real World Problems with Linear Functions (2 of 6)

Example 5:

The phone company, *Splint*, has a monthly cellular plan where the customers pay a flat monthly fee and then a cost per minute to use the phone.

If a customer uses 500 minutes, the monthly bill is $210.00. If the customer uses 610 minutes, the monthly bill is $248.50.

a. Find Splint's cost function using $C(x) = mx + b$, where x is the number of monthly minutes used, and $C(x) = y$ is the total monthly bill of the *Splint* plan.

b. If a customer's monthly bill is $321.30, how many minutes did she talk on the phone.

Model Real World Problems with Linear Functions (3 of 6)

Example 5 continued:

a. Find Splint's cost function using $C(x) = mx + b$, where x is the number of monthly minutes used, and $C(x) = y$ is the total monthly bill of the *Splint* plan.

Given the cost function $C(x) = mx + b$, we need m and b. The information given in the problem produces two ordered pairs, namely (500, 210) and (610, 248.50).

Please note that we are told the number of monthly minutes represents the independent variable x.

Model Real World Problems with Linear Functions (4 of 6)

Example 5 continued:

Given the two ordered pairs namely (500, 210) and (610, 248.50), we can calculate *m* as follows using the calculator.

$$m = \frac{248.50 - 210}{610 - 500} = 0.35$$

Since we are dealing with money, let's leave *m* in decimal form!

Model Real World Problems with Linear Functions (5 of 6)

Example 5 continued:

Next, we need b. We will use $C(x) = mx + b$, the point (500, 210) and $m = 0.35$.

$210 = 0.35(500) + b$

$210 = 175 + b$

$35 = b$

We find that Splint's cost function is $C(x) = 0.35x + 35$. This actually indicates that the flat fee Splint charges is $35 and it costs 35 cents per minute to make a phone call.

Model Real World Problems with Linear Functions (6 of 6)

Example 5 continued:

b. If a customer's monthly bill is $321.30, how many minutes did she talk on the phone?

We will use the function $C(x) = 0.35x + 35$ found in (a) and replace $C(x)$ with 321.30.

$321.30 = 0.35x + 35$

$286.30 = 0.35x$

$818 = x$

If the monthly bill is $321.30, the customer used 818 minutes talking on the phone.