# Concepts Linear Equations in Two Variables

Based on power point presentations by Pearson Education, Inc. Revised by Ingrid Stewart, Ph.D.

## Learning Objectives

- 1. Define independent and dependent variables.
- 2. Memorize the characteristics of the graphs of equations in two variables.
- 3. Define the general form of linear equations in two variables.
- 4. Graph linear equations in two variables by hand in the rectangular coordinate system using the *Point-by-Point Plotting Method* and the *Intercept Method*.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

## 1. Equations in Two Variables (1 of 2)

Up to this point, we have only been exposed to equations in one variable, for example, 6x + 3 = 0 which is a linear equation in one variable.

Now, we will discuss equations in which two variables are used, for example, 6x + 3y = 0.

**NOTE:** In mathematics we usually use the variables **x** and **y**. We then call **x** the **independent variable** and **y** the **dependent variable**.

We usually (there are some exceptions) assign numbers to the independent variable. The value of the dependent variable depends on the number we pick for the independent variable.

## Equations in Two Variables (2 of 2)

#### Example 1:

Given 6x + 3y = 0, let the independent variable x equal 1 and find the value of the dependent variable y.

Given x = 1, we calculate 6(1) + 3y = 0

then 6 + 3y = 0. This is now a linear equation in one variable, namely y.

Using the Subtraction Axiom, we get will begin to solve for y. We get 3y = -6

Finally we use the Division Axiom to get y = -2.

The value of y is -2. This value **depended** on our pick of x = 1!

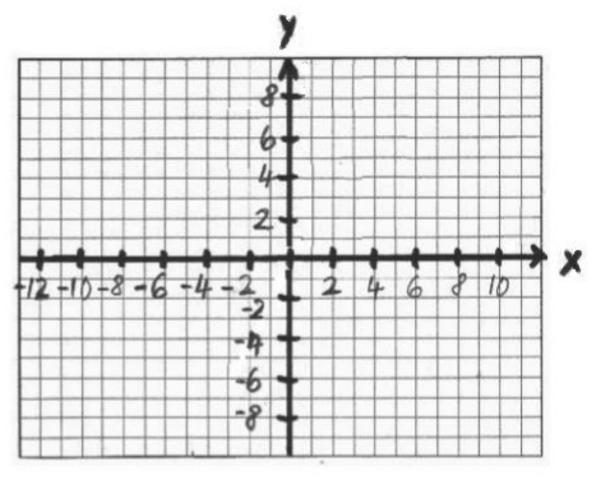
# 2. Graphs of Equations in Two Variables (1 of 7)

When we have equations in TWO variables, we can draw their pictures which are often called "graphs". We also refer to this as "graphing." Graphs of equations in two variables often give us a better understanding of some of the characteristics of the equation.

We graph equations in two variables with the help of the *Rectangular Coordinate System* in which the independent variable is assigned to the horizontal axis and the dependent variable to the vertical axis.

## Graphs of Equations in Two Variables (2 of 7)

For example, given an equation in **x** and **y** and letting **x** be the independent variable, we would call the horizontal axis in a *Rectangular Coordinate System* the **x-axis** and the vertical axis the **y-axis**. We usually label them appropriately.

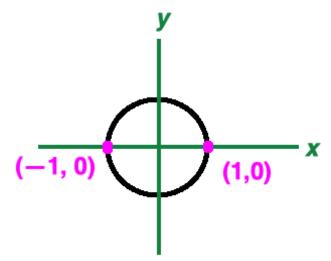


## Graphs of Equations in Two Variables (3 of 7)

Given an equation in **x** and **y**, any point in the Rectangular Coordinate System is of the form (**x**, **y**) where **x** is usually called the **x-coordinate** and **y** the **y-coordinate**.

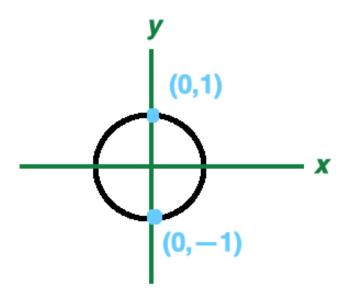
The graph of any equation in two variables, say **x** and **y**, can have **x-intercepts** and **y-intercepts**.

The **x-intercept** is the x-coordinate of the point where the graph intersects with the x-axis. The y-coordinate of this point is always 0.



## Graphs of Equations in Two Variables (4 of 7)

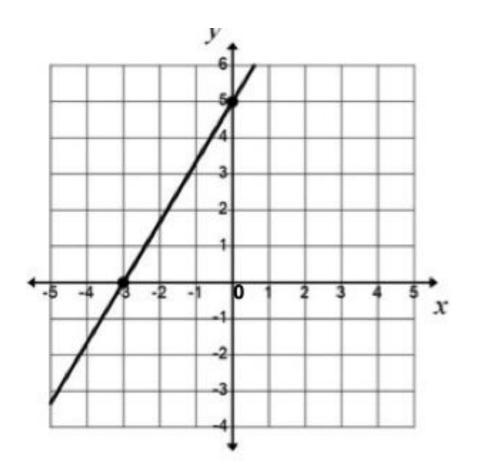
The **y-intercept** is the y-coordinate of the point where the graph intersects with the y-axis. The x-coordinate of this point is always 0.



## Graphs of Equations in Two Variables (5 of 7)

## Example 2:

Examine the graph of an equation in two variables, say x and y. Find its x-intercept and its y-intercept.



We observe that the graph crosses the x-axis at the point determined by the ordered pair (-3, 0). On the other hand, it crosses the y-axis at the point determined by the ordered pair (0, 5).

In this case, the *x*-intercept is – 3 and the *y*-intercept is 5.

## Graphs of Equations in Two Variables (6 of 7)

We try to graph equations in two variables by hand with the help of the **Point-by-Point Plotting Method** or the **Intercept Method** or a combination of both. Later we will learn other method specific to certain equations in two variables.

## When using the **Point-by-Point Plotting Method**,

- a. We pick values for the independent variable.
- b. Then we use them in the given equation to calculate corresponding values for the dependent variable.
- c. Then we create ordered pairs using the numbers in (a) and (b) above.
- d. Finally, we plot the ordered pairs from (c) above and connect the points by keeping in mind the shape of the graph.

## Graphs of Equations in Two Variables (7 of 7)

## When using the **Intercept Method**,

- a. We find the ordered pair associated with the *y*-intercept.
- b. Then we find the ordered pair associated with the x-intercept.
- c. Finally, we plot the ordered pairs from (a) and (b) above and connect the points by keeping in mind the shape of the graph.
  - NOTE: Often the *Intercept Method* will not give use enough ordered pairs to create a representative graph. When this happens, we must use the *Point-by-Point Plotting Method* or other methods to find more ordered pairs.

# 3. Linear Equations in Two Variables (1 of 3)

The general form of a linear equation in TWO variables, say x and y, is

Ax + By + C = 0, where x it the independent variable and A, B, and C are numbers, but A and B are never 0.

Incidentally, linear equations in two variables do not necessarily have to appear in *general form*. Mathematics just likes to define them that way!

## Linear Equations in Two Variables (2 of 3)

#### Examples of linear equations in two variables:

- a. 2x + 5y + 6 = 0 (general form with A = 2, B = 5, and C = 6)
- b. -18x + y + 11 = 0 (general form with A = -18, B = 1, and C = 11)
- c. x + y + 1 = 0 (general form where A = 1, B = 1, and C = 1)
- d. 9x + y = 0 (general form with A = 9, B = 1, and C = 0)

## Linear Equations in Two Variables (3 of 3)

Examples of linear equations in two variables continued:

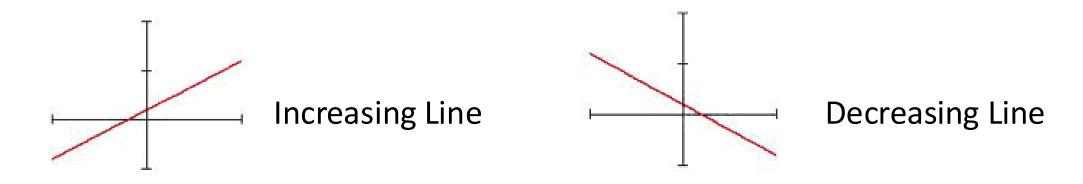
e. 
$$-4x - 6y - 7 = 0$$

Note: The *General Form* requires a plus sign between the terms. Plus signs make no changes to the equation unlike minus signs.

Please note that this equation can be written as -4x + (-6y) + (-7) = 0. Now we see that A = -4 and B = -6 and C = -7.

# 4. The Graphs of Linear Equations in Two Variables (1 of 7)

The graphs of linear equations in two variables are straight lines. These lines can be "increasing" or "decreasing" as long as  $\mathbf{A}$  and  $\mathbf{B}$  in  $\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} + \mathbf{C} = \mathbf{0}$  are not 0.



## The Graphs of Linear Equations in Two Variables (2 of 7)

When we graph linear equations in two variables by hand, we either use the **Point-by-Point Plotting Method** or the **Intercept Method** or a combination of both, to help us get started.

## The Point-by-Point Plotting Method Specific to Linear Equations:

**Step 1** – Given a linear equation in *x* and *y*, where *x* is the independent variable,

- a. We pick values for the independent variable. Usually, one to two numbers to the right and left of 0. We can also pick 0.
- b. Then we use these values in the given equation to calculate corresponding values for the dependent variable.
- c. Then we create ordered pairs using the numbers in (1) and (2) above.

## The Graphs of Linear Equations in Two Variables (3 of 7)

## Example 3:

Graph the linear equation 3x + y + 6 = 0 by hand using the *Point-by-Point Plotting Method*. Assume x is the independent variable and y the dependent variable.

Let's pick x = -4, -2, 0, and 2. You want values reasonably close to 0 or 0.

```
If x = -4, then 3(-4) + y + 6 = 0 and y = 6

If x = -2, then 3(-2) + y + 6 = 0 and y = 0

If x = 0, then 3(0) + y + 6 = 0 and y = -6

If x = 2, then 3(2) + y + 6 = 0 and y = -12
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We created the following ordered pairs:

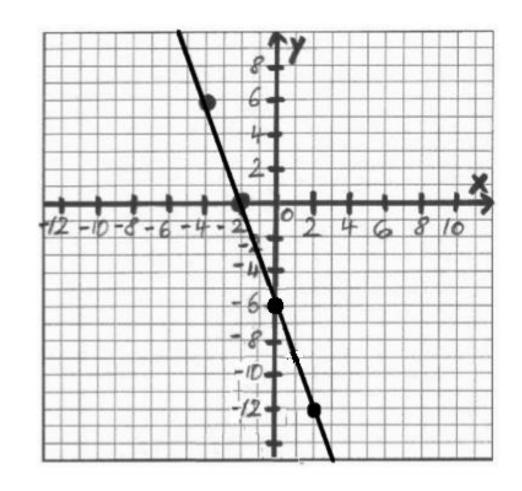
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(-4, 6) and (-2, 0) and (0, -6) and (2, -12)
```

# The Graphs of Linear Equations in Two Variables (4 of 7)

**Step 2** – Plot the ordered pairs from Step 1 in the *Rectangular Coordinate System* and connect them keeping in mind the shape of the graph of a linear equation in two variables.

Example 3 continued with 3x + y + 6 = 0:

Since we are dealing with a linear equation whose unique graph is a straight line, we will plot the ordered pairs from Step 1 and connect them with a straight line.



## The Graphs of Linear Equations in Two Variables (5 of 7)

## The Intercept Method Specific to Linear Equations:

**Step 1** – Given a linear equation in *x* and *y*, where *x* is the independent variable, **find the** *y***-intercept** and the ordered pair associated with it. We do this by **letting** *x* **be equal to 0** and solving for the dependent variable(*y*-intercept).

#### Example 4:

Graph the linear equation 3x + y + 6 = 0 by hand using the *Intercept Method*. Assume x is the independent variable and y the dependent variable!

## Let's find the y-intercept and the point associated with it:

Let x = 0 and solve for y. Then 3(0) + y + 6 = 0 and y = -6. This is the y-intercept. The ordered pair associated with the y-intercept is (0, -6).

# The Graphs of Linear Equations in Two Variables (6 of 7)

**Step 2** – Given a linear equation in *x* and *y*, where *x* is the independent variable, **find the** *x***-intercept** and the ordered pair associated with it. We do this by **letting** *y* **be equal to 0** and solving for the independent variable (*x*-intercept).

Example 4 continued with 3x + y + 6 = 0:

#### Let's find the x-intercept and the point associated with it:

Let y = 0 and solve for x. Then 3x + 0 + 6 = 0 and x = -2. This is the x-intercept! The ordered pair associated with the x-intercept is (-2, 0).

# The Graphs of Linear Equations in Two Variables (7 of 7)

**Step 3** – Plot the ordered pairs from Steps 1 and 2 in the *Rectangular Coordinate System* and connect them keeping in mind the shape of the graph of a linear equation in two variables.

Example 4 continued

$$3x + y + 6 = 0$$
:

Since we are dealing with a linear equation whose unique picture is a straight line, we will plot the ordered pairs from Steps 1 and 2 and connect them with a straight line.

