Concepts
Operations on Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Arithmetic operations on functions.
2. Composition of functions.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Arithmetic Operations on Functions (1 of 3)

When we add, subtract, multiply, and divide functions, we are doing arithmetic operations.

Let ***f*** and ***g*** be two functions. Their sum, difference, product, and quotient are also functions where the domains are the intersection of the domain of ***f*** with the domain of ***g***.

Notation:

Sum: $f(x) + g(x)$ can be expressed in function notation as $(f + g)(x)$

Difference: $f(x) - g(x)$ can be expressed in function notation as $(f - g)(x)$

Product: $f(x)\,g(x)$ can be expressed in function notation as $(f\,g)(x)$

Quotient: $\frac{f(x)}{g(x)}$ can be expressed in function notation as $\left(\frac{f}{g}\right)(x)$ *where* $g \neq 0$

Arithmetic Operations on Functions (2 of 3)

Example 1:

a. Let $f(x) = -2x^2$ and $g(x) = 5x^2 + 3$. Find the functions ***(f + g)(x)*** and ***(f – g)(x)*** and simplify, if necessary.

$$\begin{aligned}(f+g)(x) &= (-2x^2) + (5x^2 + 3) \\ &= -2x^2 + 5x^2 + 3 \\ &= 3x^2 + 3\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= (-2x^2) - (5x^2 + 3) \\ &= -2x^2 - 5x^2 - 3 \\ &= -7x^2 - 3\end{aligned}$$

Arithmetic Operations on Functions (3 of 3)

Example 1 continued:

b. Let $f(x) = -2x^2$ and $g(x) = 5x^2 + 3$. Find the functions $\boldsymbol{(f\,g)(x)}$ and $\left(\frac{f}{g}\right)(x)$ and simplify, if necessary.

$(f\,g)(x) = (-2x^2)(5x^2 + 3) = -10x^4 - 6x^2$ used the *Distributive Property* and the *Product Rule of exponents*!)

$\left(\frac{f}{g}\right)(x) = \frac{-2x^2}{5x^2+3}$ where the denominator $5x^2 + 3$ cannot be equal to 0!

2. Composition of Functions (1 of 4)

Given the functions ***f*** and ***g,*** we can form their composites in two different ways. We can find the composition of the function ***f*** with ***g*** and the composition of the function ***g*** with ***f***.

The notation for the composition of the function ***f*** with ***g*** is $f \circ g$. We read this as "***f*** composed of ***g*** ".

The notation for the composition of the function ***g*** with ***f*** is $g \circ f$. We read this as "***g*** composed of ***f*** ".

NOTE: The symbol for function composition is a circle ∘ and NOT A DOT! Please do not confuse it with a multiplication symbol!

Composition of Functions (2 of 4)

Given the independent variable *x*, the notation ***(f ∘ g)(x)*** is read as "***f*** composed of ***g*** of ***x*** ". It means ***f [g(x)]*** which is read as "***f*** of ***g*** of ***x*** ".

We find ***f [g(x)]*** by replacing every occurrence of ***x*** in the function ***f*** with the function ***g***.

Given the independent variable *x,* the notation ***(g ∘ f)(x)*** is read as "***g*** composed of ***f*** of ***x*** ". It means ***g [f (x)]*** which is read as "***g*** of ***f*** of ***x*** ".

We find ***g [f (x)]*** by replacing every occurrence of ***x*** in the function ***g*** with the function ***f***.

Please note that the results of ***f ∘ g*** and ***g ∘ f*** are often NOT equal, but they can be.

Composition of Functions (3 of 4)

Example 2:

a. Given $f(x) = \sqrt[3]{x} - 1$ and $g(x) = x^3$, find $(f \circ g)(x)$ and simplify, if necessary.

We find $(f \circ g)(x)$ by replacing every occurrence of x in the function f with the function g as follows:

$$(f \circ g)(x) = f[g(x)] = f(x^3)$$

$$= \sqrt[3]{x^3} - 1$$

This is the function f where x was replaced by x^3.

$$= x - 1$$

Composition of Functions (4 of 4)

Example 2 continued:

b. Given $f(x) = \sqrt[3]{x} - 1$ and $g(x) = x^3$, find $(g \circ f)(x)$. No need to simplify.

We find $(g \circ f)(x)$ by replacing every occurrence of x in the function g with the function f as follows:

$$(g \circ f)(x) = g[f(x)] = g(\sqrt[3]{x} - 1)$$

$$= (\sqrt[3]{x} - 1)^3$$

This is the function g where x was replaced by $\sqrt[3]{x} - 1$.

Please note that the results of $f \circ g$ and $g \circ f$ are not the same!