DETAILED SOLUTIONS AND CONCEPTS
GRAPHS OF TANGENT, COTANGENT, SECANT, AND COSECANT FUNCTIONS
Prepared by Ingrid Stewart, Ph.D., College of Southern Nevada
Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

Definition of the Tangent Function

$$f(x) = \tan x$$

- The *Tangent Function* is called "periodic" with *period* $\pi \approx 3.14$.
- The domain contains all real numbers except $\pi/2 + k\pi$, where k is any integer, which is can be expressed as $\{x \mid x \neq \frac{\pi}{2} + k\pi\}$ in *Set-Builder Notation*.
- At the values that are excluded from the domain, the graph of the *Tangent Function* has *vertical asymptotes*. The equations of the *vertical asymptotes* are $x = \pi/2 + k\pi$.

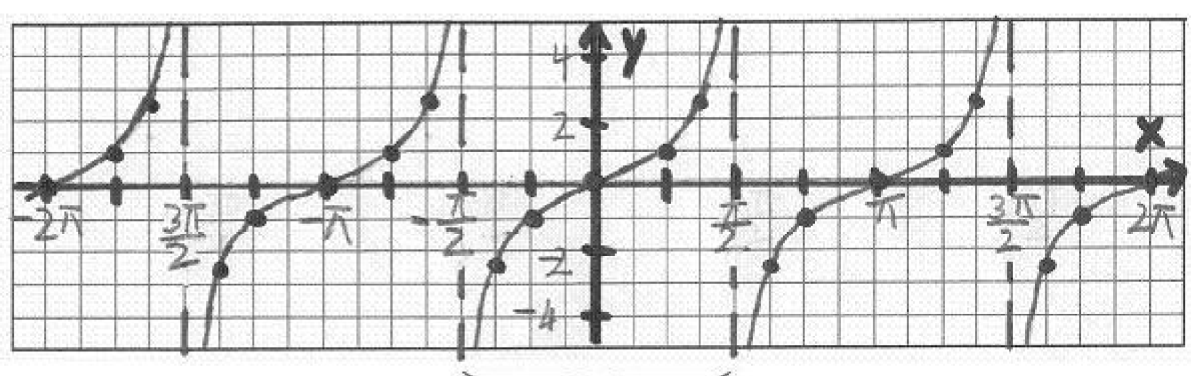
Characteristics of Graphs of the Tangent Function

You can either take my word that the graph of the *Tangent Function* appears as below or you can convince yourself by using point-by-point plotting!

Please observe the following:

Given $f(x) = \tan x$, we can create the following table of numbers:

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
$\tan x$	-1	0	1



Representative Picture

Now let's look at the graph above and make some observations:

a. The ***representative picture*** of the graph of $f(x) = \tan x$ lies on the interval $(-\pi/2, \pi/2)$. It repeats itself "forever" along the positive and negative x-axis.

Please note that each branch is concave up  **to the right of each x-intercept and concave down**  **to the left.**

b. Obviously then, it takes a distance of $\pi \approx 3.14$ along the x-axis for the *representative picture* to be created. This distance is again called the ***period***, which is considered one ***cycle*** of the tangent function.

c. The representative picture appears to be divided into **four equal intervals**. Yes, this seems to be happening here also! It's just not as obvious as it was for the graphs of the sine and cosine functions!

d. The *representative picture* lies in between the vertical asymptotes $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

e. The *representative picture* never touches the asymptotes, but is also never parallel to them.

f. An x-intercept lies exactly in the middle of the period.

Definition of the Cotangent Function

$$g(x) = \cot x$$

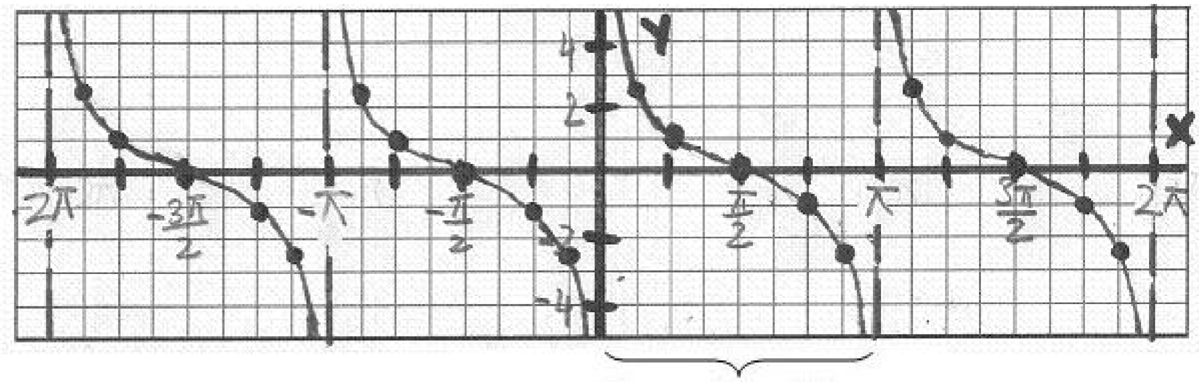
- The *Cotangent Function* is called "periodic" with *period* $\pi \approx 3.14$.
- The domain contains all real numbers except $k\pi$, where k is any integer, which is can be expressed as $\{x \mid x \neq k\pi\}$ in *Set-Builder Notation*.
- At the values that are excluded from the domain, the graph of the *Cotangent Function* has *vertical asymptotes*. The equations of the *vertical asymptotes* are $x = k\pi$.

Characteristics of Graphs of the Cotangent Function

Again, you can either take my word that the graph of the *Cotangent Function* appears as below or you can convince yourself by using point-by-point plotting!

For example,

x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\cot x$	1	0	-1



Representative Picture

Let's look at the graph above and make some observations:

a. The ***representative picture*** of the graph of $g(x) = \cot x$ lies on the interval ***(0, π)***. It repeats itself "forever" along the positive and negative x-axis.

Please note that each branch is concave up  **to the left of each x-intercept and concave down**  **to the right.**

b. Obviously then, it takes a distance of $\pi \approx 3.14$ along the x-axis for the *representative picture* to be created. This distance is again called the ***period***, which is considered one ***cycle*** of the tangent function.

c. The representative picture appears to be divided into **four equal intervals**. Yes, this seems to be happening here also! It's just not as obvious as it was for the graphs of the sine and cosine functions!

d. The *representative picture* lies in between the vertical asymptotes $x = 0$ and $x = \pi$.

e. The *representative picture* never touches the asymptotes, but is also never parallel to them.

f. An x-intercept lies exactly in the middle of the period.

Strategy for Graphing $f(x) = \tan x$ and $g(x) = \cot x$ - You must memorize this!

NOTE: To produce graphs of trigonometric functions with a hand-held graphing calculator, be sure to switch to radian mode!!!

1. Graph the *representative picture*.

   - Draw a *Cartesian Coordinate System* showing a horizontal and vertical axis. Most likely you will use an x-axis and a y-axis.
   - Mark off a distance along the x-axis to represent the *period* π.
   - Draw dashed vertical lines at the beginning and end of the *period* to represent the *vertical asymptotes*.

NOTE: If the y-axis is a vertical asymptote, it is customary NOT to use a dashed line!

- Divide the *period* into four equal intervals.

- For the beginning and ending point of each interval within a *period*, calculate the y-value and plot the points. This will give you your x-intercept and two additional points to the right and left of the x-intercept.

  Remember, the beginning and ending point of each interval within a *period* have a certain x-value and given $y = f(x)$ or $y = g(x)$ you can then find the corresponding y-value.

- Find at least two additional points closer to the asymptotes within each period!

- Connect the points to form either the *representative picture* of the tangent or cotangent function.

2. Copy several more cycles in the same manner to the right and left of the *representative picture*.

Transformations of the Graphs of the Tangent and Cotangent Functions

Previously, we discussed the functions $f(x) = \tan x$ and $g(x) = \cot x$ and how to graph them. Let's call them BASIC FUNCTIONS.

Now we will learn how to graph transformations of these basic functions, specifically of the form

$f(x) = a\tan(bx)$ and $g(x) = a\cot(bx)$, where a and b can take on any value except 0

NOTE: These are the only transformations we will be investigating in detail in this course.

- The value a indicates a **vertical stretch or shrink** in the graph of the basic functions. This will change the y-values of the basic functions.

  In the case of tangent and cotangent functions, a does NOT represent an amplitude because both functions are considered continuously increasing.

- The value b indicates a **horizontal stretch or shrink** in the graph of the basic functions. This will affect the size of the period of the basic functions.

  The formula $P = \frac{\pi}{b}$ describes the period P of the graph of a trigonometric function.

Strategy for Graphing the Transformations

- Use the strategy as outlined above for the basic functions $f(x) = \tan x$ and $g(x) = \cot x$.
- The only difference might be that you may have to use the the formula $P = \frac{\pi}{b}$ to calculate the period.

Following is a discussion of the graphs of the *Cosecant* and *Secant Functions*. You will not have to graph them by hand. However, you must have at least a general idea of what their graphs look like.

In this course, you will be asked to graph these two functions with the help of a computer graphing utility.

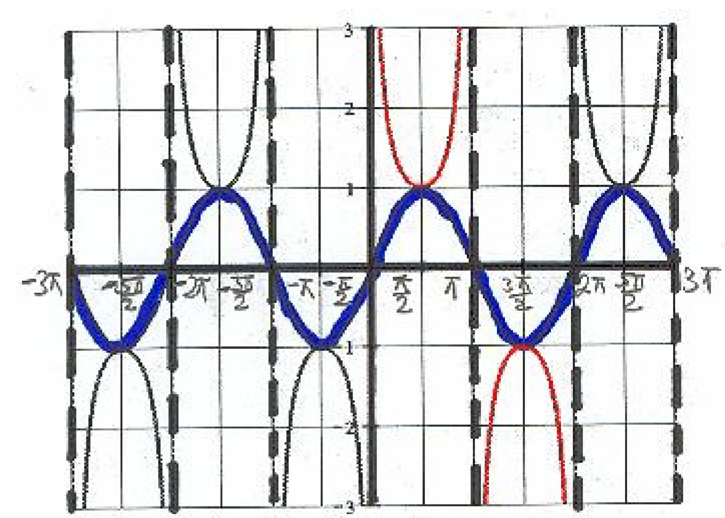
Definition of the Cosecant Function

$h(x) = \csc x$

- The cosecant function is called "periodic" with *period* $2\pi \approx 6.28$.
- Domain: ***All Real Numbers*** except $k\pi$ where k is any integer.
- At the values that are excluded from the domain, the graph of the *Cosecant Function* has *vertical asymptotes*. The equations of the *vertical asymptotes* are $x = k\pi$.

Below is the graph of the *Cosecant Function* together with its reciprocal, the *Sine Function*.

You can either take my word that the graph of the cosecant function appears as below or you can convince yourself by using point-by-point plotting using many, many more points than I used!



Please observe the following:

- The *representative picture* of the cosecant function lies on the interval ***(0, 2π)***.
- Each parabola-like graph is separated by a *vertical asymptote* ***, which occur at the x-intercepts of the sine function.
- The *representative picture* never touches the asymptotes, but is also never parallel to them.
- There are neither x- nor y-intercepts.
- The graph of the cosecant function has its valleys where the graph of the sine function has peaks, and vice versa.

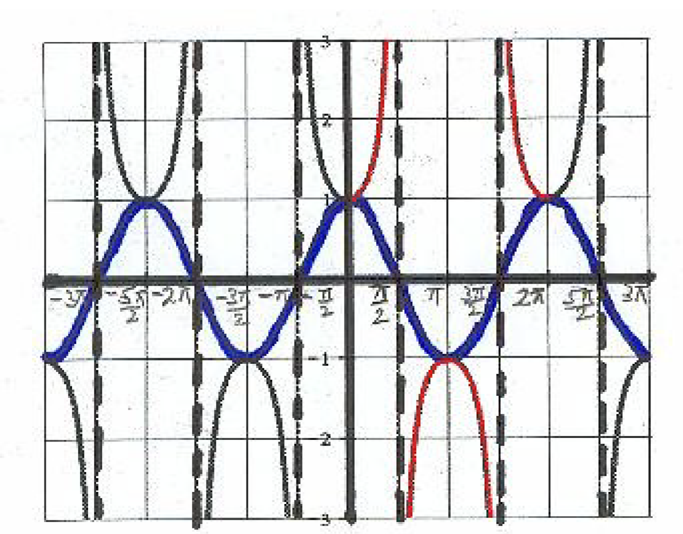
Definition of the Secant Function

$$p(x) = \sec x$$

- The secant function is called "periodic" with *period* $2\pi \approx 6.28$.
- Domain: ***All Real Numbers*** except $\pi/2 + k\pi$ where k is any integer.
- At the values that are excluded from the domain, the graph of the *Secant Function* has *vertical asymptotes*. The equations of the *vertical asymptotes* are $x = \pi/2 + k\pi$.

Below is the graph of the *Secant Function* together with its reciprocal, the *Cosine Function*.

Again, you can either take my word that the graph of the cosecant function appears as below or you can convince yourself by using point-by-point plotting using many, many more points than I used!



Please observe the following:

- The *representative picture* of the secant function lies on the interval ***[0, 2π]***.
- Each parabola-like graph is separated by a *vertical asymptote*. The *vertical asymptotes* occur at the x-intercepts of the cosine function.
- The *representative picture* never touches the asymptotes, but is also never parallel to them.
- There are no x-intercepts, but one y-intercept.
- The graph of the secant function has its valleys where the graph of the cosine function has peaks, and vice versa.

Problem 1:

Given $f(x) = 2\cot(2x)$, do the following:

a. state the **EXACT** period
b. graph the function on the interval $(-\pi, \pi)$
c. find the equations of the *vertical asymptotes* using the graph
d. find the **EXACT** coordinates of the x-intercepts using the graph
e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph
f. find $f(\pi/6)$ rounded to four decimal places

a. state the **EXACT** period

The period is $\pi/2$. This means the representative picture is $\pi/2$ units in length.

NOTE: To find an **EXACT** period means that we want to express it in terms of π and not use a decimal approximation such as $\pi \approx 3.14$.

b. graph the function on the interval $(-\pi, \pi)$

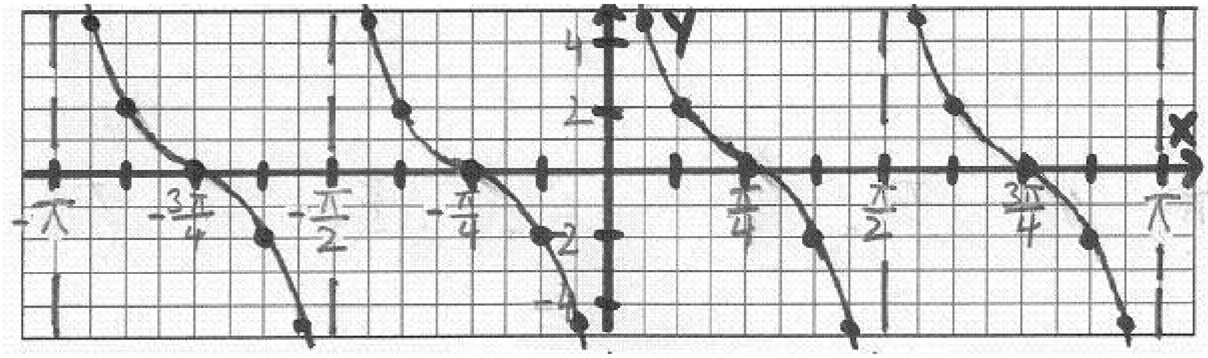
Graph the representative picture.

- Mark off a distance along the x-axis starting at the origin to represent the period $\pi/2$.
- Divide the period into four equal intervals of length $\pi/8$.
- Draw dashed vertical lines at the beginning and end of the period to represent the *vertical asymptotes* (do not draw the y-axis as a dashed line).
- Find the y-values of the ending points of the other intervals and plot the points.
- Connect the points to form the representative picture of the cotangent function observing the various concavities of the graph!

Copy several more cycles in the same manner to the right and left of the representative picture on the interval $(-\pi, \pi)$.

1. **Please note the concavities of the graphs of the tangent and cotangent functions.**

2. **Also, each branch continues to head toward the vertical asymptotes without ever "getting there." Do not draw the branches parallel to the asymptotes!**



c. find the equations of the *vertical asymptotes* using the graph

Remember that each period is divided into four equal intervals of length $\pi/8$!

$x = -\pi$ $\quad x = -\pi/2$ $\quad x = 0$ $\quad x = \pi/2$ $\quad x = \pi$

d. find the **EXACT** coordinates of the x-intercepts using the graph

> NOTE: To find the **EXACT** coordinates means that we want to express them in terms of π and not use a decimal approximation such as $\pi \approx 3.14$.

Remember that each period is divided into four equal intervals of length $\pi/8$!

$(-3\pi/4, 0)$, $(-\pi/4, 0)$, $(\pi/4, 0)$, $(3\pi/4, 0)$

e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph

Remember that each period is divided into four equal intervals of length $\pi/8$!

$(-7\pi/8, 2)$, $(-5\pi/8, -2)$, $(-3\pi/8, 2)$, $(-\pi/8, -2)$,

$(\pi/8, 2)$, $(3\pi/8, -2)$, $(5\pi/8, 2)$, $(7\pi/8, -2)$

f. Find $f(\pi/6)$ rounded to two decimal places.

2	*cos*	*(*	*2*	*	π	/	*6*	*)*	/	*sin*	*(*	*2*	*	π	/	*6*	*)*	*Enter*

$f(\pi/6) \approx 1.15$

Problem 2:

Given is the function $g(x) = 2\tan\left(\frac{\pi x}{4}\right)$, do the following:

a. state the **EXACT** period
b. graph the function on the interval ***[-8, 8]***
c. find the equations of the *vertical asymptotes* using the graph
d. find the **EXACT** coordinates of the x-intercepts using the graph
e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph
f. find $g(\pi/6)$ rounded to four decimal places

a. state the **EXACT** period

The period is the period equals $\frac{\pi}{\frac{\pi}{4}} = \pi \cdot \frac{4}{\pi} = 4$. This means the representative picture is ***4*** units in length.

b. graph the function on the interval ***[-8, 8]***

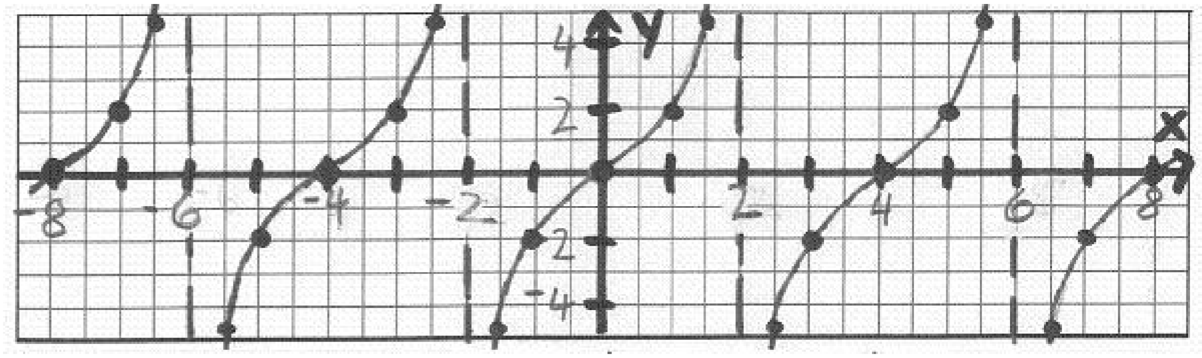
Graph the representative picture.

- Mark off a distance along the x-axis to represent the period ***4***. Note that the representative picture of the tangent function is bisected by the y-axis!
- Divide the period into four equal intervals of length ***1***.
- Draw dashed vertical lines at the beginning and end of the period to represent the *vertical asymptotes*.
- Find the y-values of the ending points of the other intervals and plot the points.
- Connect the points to form the representative picture of the tangent function observing the various concavities of the graphs!

Copy several more cycles in the same manner to the right and left of the representative picture on the interval ***[-8, 8]***.

1. **Please note the concavities of the graphs of the tangent and cotangent functions.**

2. **Also, each branch continues to head toward the vertical asymptotes without ever "getting there." Do not draw the branches parallel to the asymptotes!**



c. find the equations of the *vertical asymptotes* using the graph

Remember that each period is divided into four equal intervals each of length ***1***!

x = -6 x = -2 x = 2 x = 6

d. find the **EXACT** coordinates of the x-intercepts using the graph

Remember that each period is divided into four equal intervals each of length ***1***!

(-8, 0), (-4, 0), (0, 0), (4, 0), (8, 0)

e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph

Remember that each period is divided into four equal intervals of length ***1***!

(-7, 2), (-5, -2), (-3, 2), (-1, -2), (1, 2), (3, -2), (5, 2), (7, -2)

f. find $g(\pi/6)$ rounded to three decimal places.

2	***tan***	***(***	***(***	π	/	***4***	***)***	***(***	π	/	***6***	***)***	***)***	***Enter***

$g(\pi/6) \approx 0.872$

Problem 3:

Given is the function $y = 2\tan(4x)$, do the following:

a. state the **EXACT** period

b. graph the function on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

c. find the equations of the *vertical asymptotes* using the graph

d. find the **EXACT** coordinates of the x-intercepts using the graph

e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph

a. state the **EXACT** period

The period is the period equals $\pi/4$. This means the representative picture is 4 units in length.

b. graph the function on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

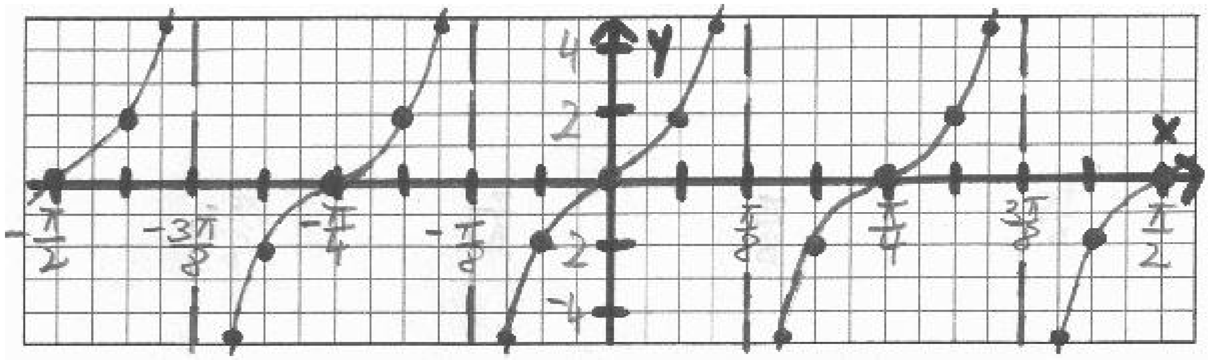
Graph the representative picture.

- Mark off a distance along the x-axis to represent the period $\pi/4$. Note that the representative picture of the tangent function is bisected by the y-axis!
- Divide the period into four equal intervals of length $\pi/16$.
- Draw dashed vertical lines at the beginning and end of the period to represent the *vertical asymptotes*.
- Find the y-values of the ending points of the other intervals and plot the points.
- Connect the points to form the representative picture of the tangent function observing the various concavities of the graphs!

Copy several more cycles in the same manner to the right and left of the representative picture on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

1. **Please note the concavities of the graphs of the tangent and cotangent functions.**

2. **Also, each branch continues to head toward the vertical asymptotes without ever "getting there." Do not draw the branches parallel to the asymptotes!**



c. find the equations of the *vertical asymptotes* using the graph

Remember that each period is divided into four equal intervals each of length ***π/16***!

x = -3π/8 x = -π/8 x = 0 x = π/8 x = 3π/8

d. find the **EXACT** coordinates of the x-intercepts using the graph

Remember that each period is divided into four equal intervals each of length ***π/16***!

(-π/2, 0), (-π/4, 0), (0, 0), (π/4, 0), (π/2, 0)

e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph

Remember that each period is divided into four equal intervals of length ***1***!

(-7π/16, 2), (-5π/16, -2), (-3π/16, 2), (-π/16, -2)

(π/16, 2), (3π/16, -2), (5π/16, 2), (7π/16, -2)

Problem 4:

Given $y = cot\left(\frac{\pi x}{3}\right)$, do the following:

a. state the **EXACT** period
b. graph the function on the interval ***(-6, 6)***
c. find the equations of the *vertical asymptotes* using the graph
d. find the **EXACT** coordinates of the x-intercepts using the graph
e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph

a. state the **EXACT** period

The period is $\frac{\pi}{\frac{\pi}{3}} = \pi \cdot \frac{3}{\pi} = 3$. This means the representative picture is ***3*** units in length.

NOTE: To find an **EXACT** period means that we want to express it in terms of π and not use a decimal approximation such as $\pi \approx 3.14$.

b. graph the function on the interval ***(-6, 6)***

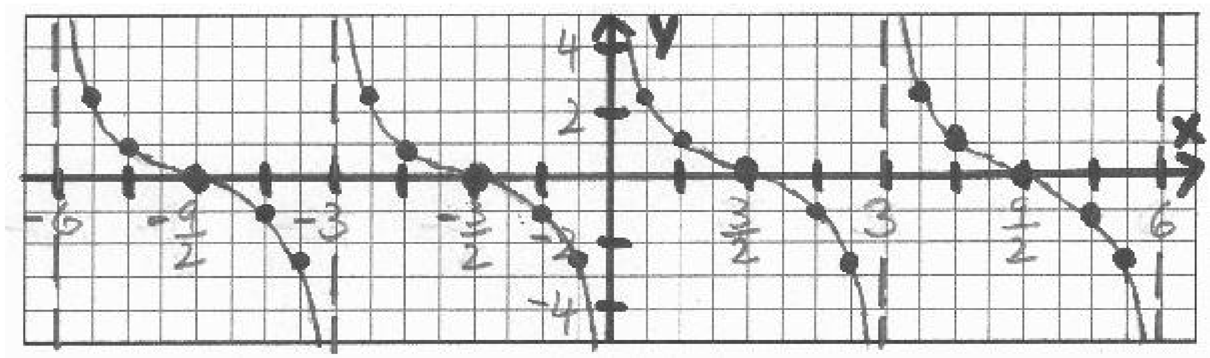
Graph the representative picture.

- Mark off a distance along the x-axis starting at the origin to represent the period ***3***.
- Divide the period into four equal intervals of length ***3/4***.
- Draw dashed vertical lines at the beginning and end of the period to represent the *vertical asymptotes* (do not draw the y-axis as a dashed line).
- Find the y-values of the ending points of the other intervals and plot the points.
- Connect the points to form the representative picture of the cotangent function observing the various concavities of the graph!

Copy several more cycles in the same manner to the right and left of the representative picture on the interval ***(-6, 6)***.

1. **Please note the concavities of the graphs of the tangent and cotangent functions.**

2. **Also, each branch continues to head toward the vertical asymptotes without ever "getting there." Do not draw the branches parallel to the asymptotes!**



c. find the equations of the *vertical asymptotes* using the graph

Remember that each period is divided into four equal intervals of length π ***/8***!

x = -6 ***x = -3*** ***x = 0*** ***x = 3*** ***x = 6***

d. find the **EXACT** coordinates of the x-intercepts using the graph

NOTE: To find the **EXACT** coordinates means that we want to express them in terms of π and not use a decimal approximation such as $\pi \approx$ ***3.14***.

Remember that each period is divided into four equal intervals of length π ***/8***!

(-9/2, 0), (-3/2, 0), (3/2, 0), (9/2, 0)

e. find the **EXACT** coordinates of the points at the end of the first and third interval of each period using the graph

Remember that each period is divided into four equal intervals of length π ***/8***!

(21/4, 1), (-15/4, -1), (-9/4, 1), (-3/4, -1)

(3/4, 1), (9/4, -1), (15/4, 1), (21/4, -1)