PROBLEMS AND SOLUTIONS - RATIONAL FUNCTIONS

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Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

Problem 1:

Given the rational function $f(x) = \frac{1}{x}$, which is also called the **Reciprocal Function**, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 2:

Given the rational function $f(x) = \frac{4x}{2x^2 + 1}$, find the following:

a. the Domain in Interval Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 3:

Given the rational function $g(x) = \frac{4x^2}{x^2 - 1}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 4:

Given the rational function $h(x) = \frac{4x^5 + 11x^2}{2x^2 + 1}$, find the following:

a. the Domain in Interval Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 5:

Given the rational function $f(x) = \frac{2x^2 + 9x - 6}{x^2 + 2x - 3}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 6:

Given the rational function $f(x) = \frac{4x^2}{2x^2 + 1}$, find the following:

a. the Domain in Interval Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 7:

Given the rational function $f(x) = \frac{-x^2 - 4x}{(x+2)^2}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 8:

Given the rational function $p(x)=\dfrac{x^2-3}{2x-4}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 9:

Given the rational function $f(x)=\dfrac{x+2}{x^2-4}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 10:

Given the rational function $f(x)=\dfrac{x^2+2x-3}{2x^2-12x+10}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 11:

Given the rational function $k(x)=\dfrac{x-1}{x^2-1}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

Problem 12:

Given the rational function $h(x) = \dfrac{x^2 - 1}{x - 1}$, find the following:

a. the Domain in Set-Builder Notation
b. the Equation of any Vertical Asymptotes
c. the Equation of any Horizontal Asymptotes
d. the Equations of any Oblique Asymptotes
e. the Coordinates of any Holes

SOLUTIONS

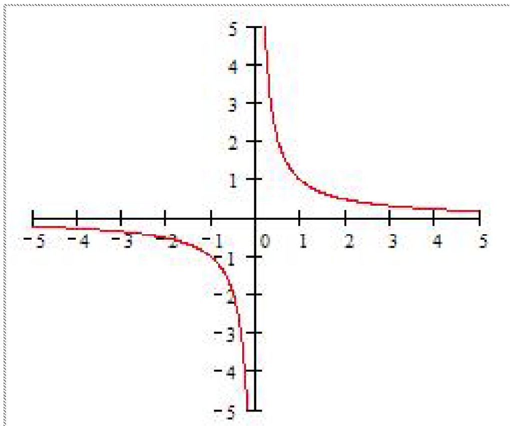
You can find detailed solutions below the link for this problem set!

Problem 1:

- Domain: $\{x \mid x \neq 0\}$
- Equation of the *Vertical Asymptote:* ***x = 0***
- Equation of the *Horizontal Asymptote*: ***y = 0***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***None***

Below is the graph of the function.

Please note that the branches of rational functions have SMOOTH turns. They are never parallel to their asymptotes, but move toward them at a steady pace.

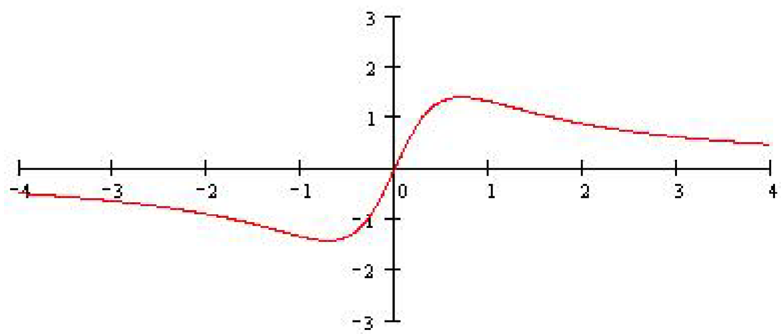


Please note that the x-axis is a *horizontal asymptote* and the y-axis is the *vertical asymptote.*

Problem 2:

- Domain: $(-\infty, \infty)$
- Equation of the *Vertical Asymptote:* ***None***
- Equation of the *Horizontal Asymptote*: ***y = 0***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***None***

Below is the graph of the function.

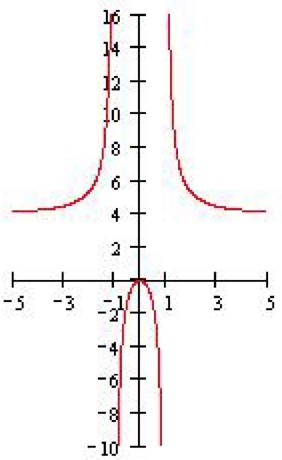


Please note that the x-axis is a *horizontal asymptote*. There are no *vertical asymptotes*! Remember that graphs may cross/touch their *horizontal asymptotes*!

Problem 3:

- Domain: $\{x \mid x \neq -1, x \neq 1\}$
- Equation of the *Vertical Asymptote:* $x = -1$ and $x = 1$
- Equation of the *Horizontal Asymptote*: ***y = 4***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***None***

Below is the graph of the function.

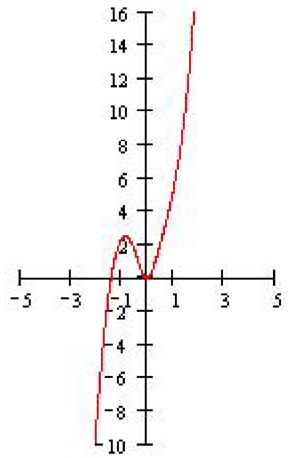


There is a *horizontal asymptote* at *y = 4* and two *vertical asymptotes* at $x = -1$ and $x = 1$. Please note that asymptotes are invisible lines and NO graphing program places lines into a graph to represent asymptotes.

Problem 4:

- Domain: $(-\infty, \infty)$
- Equation of the *Vertical Asymptote:* ***None***
- Equation of the *Horizontal Asymptote*: ***None***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***None***

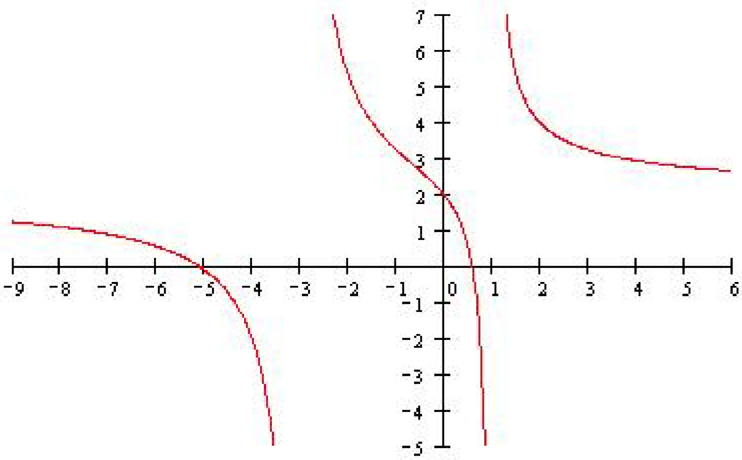
Below is the graph of the function.

	Please note that there are no asymptotes!

Problem 5:

- Domain: $\{x \mid x \neq -3, x \neq 1\}$
- Equation of the *Vertical Asymptote:* $x = -3$ and $x = 1$
- Equation of the *Horizontal Asymptote*: ***y = 2***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***None***

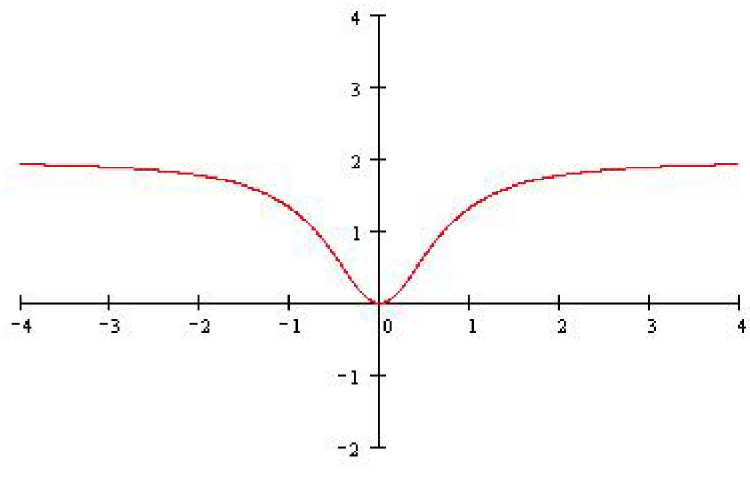
Below is the graph of the function.

	There is a *horizontal asymptote* at $y = 2$ and two *vertical asymptotes* at $x = -3$ and $x = 1$. Remember that graphs may cross/touch their *horizontal asymptotes*!

Problem 6:

- Domain: $(-\infty, \infty)$
- Equation of the *Vertical Asymptote:* ***None***
- Equation of the *Horizontal Asymptote*: ***y = 2***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***None***

Below is the graph of the function.

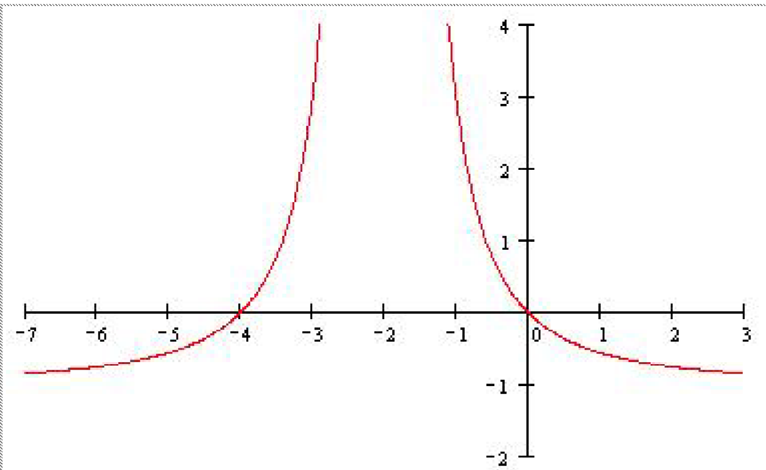


There is a *horizontal asymptote* at $y = 2$ and no *vertical asymptotes*.

Problem 7:

- Domain: $\{x \mid x \neq -2\}$
- Equation of the *Vertical Asymptote:* $x = -2$
- Equation of the *Horizontal Asymptote*: $y = -1$
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***None***

Below is the graph of the function.

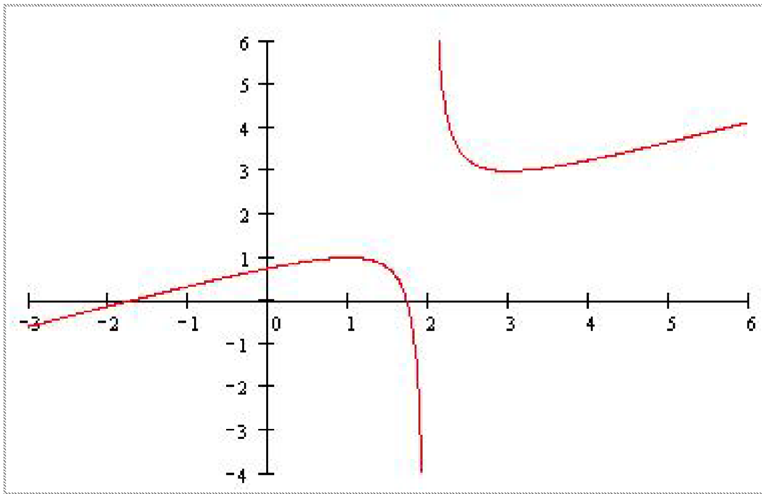


There is a *horizontal asymptote* at $y = -1$ and a *vertical asymptote* at $x = -2$.

Problem 8:

- Domain: $\{x \mid x \neq 2\}$
- Equation of the *Vertical Asymptote:* $x = 2$
- Equation of the *Horizontal Asymptote*: ***None***
- Equation of the *Oblique Asymptote:* $y = \frac{1}{2}x + 1$
- Coordinates of any Holes: ***None***

Below is the graph of the function.

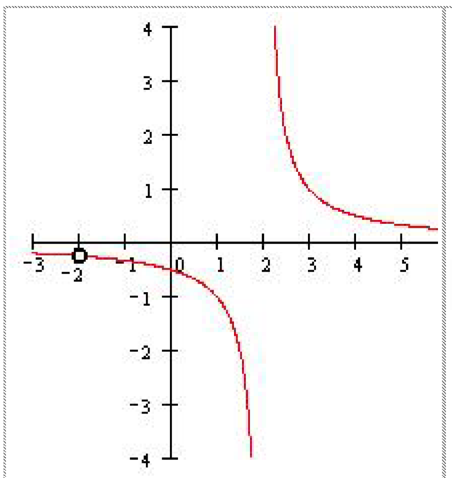


There is a *vertical asymptote* at $x = 2$ and an *oblique asymptote* at $y = \frac{1}{2}x + 1$.

Problem 9:

- Domain: $\{x \mid x \neq -2, x \neq 2\}$
- Equation of the *Vertical Asymptote:* $x = 2$
- Equation of the *Horizontal Asymptote*: $y = 0$
- Equation of the *Oblique Asymptote:* **None**
- Coordinates of any Holes: $(-2, -\frac{1}{4})$

Below is the graph of the function.

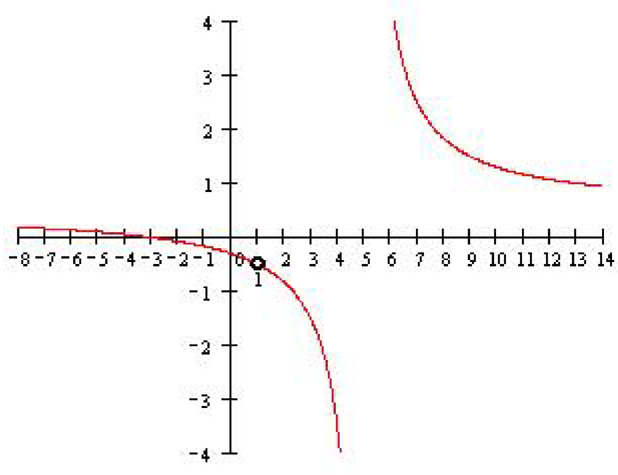


Please note that the x-axis is a *horizontal asymptote*. There is also a *vertical asymptote* at $x = 2$. Please observe the hole in the graph at $(-2, -\frac{1}{4})$.

Problem 10:

- Domain: $\{x \mid x \neq 1, x \neq 5\}$
- Equation of the *Vertical Asymptote:* $x = 5$
- Equation of the *Horizontal Asymptote*: $y = \frac{1}{2}$
- Equation of the *Oblique Asymptote:* **None**
- Coordinates of any Holes: $(1, -\frac{1}{2})$

Below is the graph of the function.

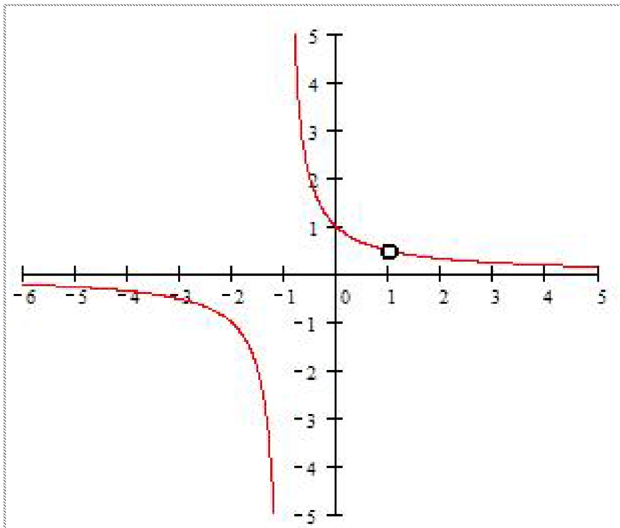


There is a *horizontal asymptote* at $y = \frac{1}{2}$ and a *vertical asymptote* at $x = 5$. Please observe the hole in the graph at $(1, -\frac{1}{2})$.

Problem 11:

- Domain: $\{x \mid x \neq -1, x \neq 1\}$
- Equation of the *Vertical Asymptote:* ***x = -1***
- Equation of the *Horizontal Asymptote*: ***y = 0***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: $(1, \frac{1}{2})$

Below is the graph of the function.

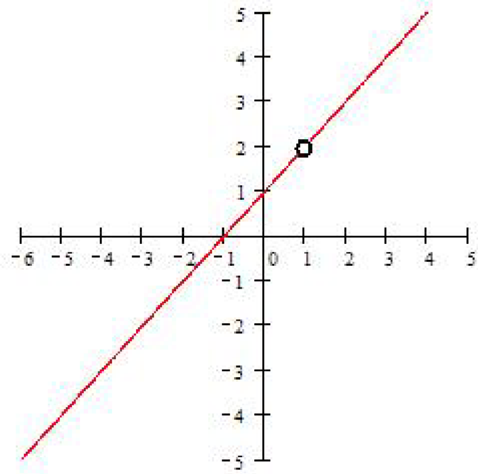


There is a *horizontal asymptote* at ***y = 0*** and a *vertical asymptote* at ***x = -1***. Please observe the hole in the graph at $(1, \frac{1}{2})$.

Problem 12:

- Domain: $\{x \mid x \neq 1\}$
- Equation of the *Vertical Asymptote:* ***None***
- Equation of the *Horizontal Asymptote*: ***None***
- Equation of the *Oblique Asymptote:* ***None***
- Coordinates of any Holes: ***(1, 2)***

Below is the graph of the function.



There is NO *horizontal* or *vertical asymptote*. Please observe the hole in the graph at ***(1, 2)***.