DETAILED SOLUTIONS AND CONCEPTS - DECIMALS AND WHOLE NUMBERS

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YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

There are many different types of numbers. The three types we will be examining in this lecture are the **Natural Numbers**, the **Whole Numbers**, and **Decimal Numbers** also called **Decimals**.

NATURAL NUMBERS

The *Natural Numbers*, sometimes called *Counting Numbers*, begin with the number ***1*** and continue indefinitely without end. The term that is used to describe this phenomenon is **infinity** and its symbol is ∞.

{1, 2, 3, 4, 5, ...}

When a number has more than one digit, each digit is assigned a **place** or **place value**.

Example 1:

The number *thirty-five thousand four hundred and twelve*, ***35412***, has five digits. Their respective places or place values are as follows:

3 is in the **ten thousands** place
5 is in the **thousands** place
4 is in the **hundreds** place
1 is in the **tens** place
2 is in the **ones** place

Example 2:

35412 can be written as ***35,412***

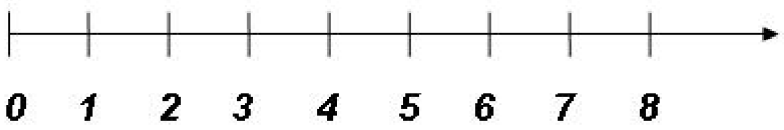
Often a comma is used to separate larger numbers into groups of three digits counting from right to left. This is optional and used for easier reading of the number.

WHOLE NUMBERS

The *Whole Numbers* include all of the *Natural Numbers* and the number ***0***.

{0, 1, 2, 3, 4, 5, ...}

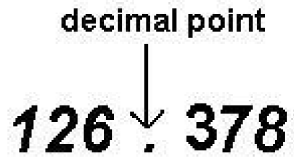
Whole Numbers can be arranged on a number line to show a visual representation of the relationship of their size.



DECIMAL NUMBERS

Our counting system lets us write fractional amounts of whole numbers using a clever symbol called the **decimal point**. When a number contains a decimal point, we call it a **decimal number** or simply a **decimal**.

Example 1:



Place value of the digits of the whole number part of the decimal.

1 is in the **hundreds** place
2 is in the **tens** place
6 is in the **ones** place

Place value of the digits of the fractional part of the decimal. This is usually referred to as the **decimal place**.

3 is in the **tenths** place
7 is in the **hundredth** place
8 is in the **thousandths** place

NOTE: The number ***126.378*** is most often read as **"one hundred twenty six point three seven eight."**

Example 2:

Write the whole number ***45*** as a decimal number.

45.0 or ***45.00*** or ***45.000*** etc.

Note that you can add as many decimal places as you need depending on a particular situation.

Example 3:

In some decimal numbers the whole number part is zero. This is either indicated by a zero to the left of the decimal point or the whole number place is simply left blank.

0.0024 or ***.0024***

ADDITION

Vocabulary:

In ***8 + 2 = 10***, the ***10*** is called **Sum.**

Rule for Adding Whole Numbers:

- Align the numbers in columns by place value.
- Then add the digits in each column starting on the right.
- Regroup whenever the sum of a column is more than one digit.

Problem 1:

Find the sum of ***180 + 87 + 15*** without a calculator.

```
 180
  87
+ 15
 11
----
 282
```

- The sum of the digits in the *ones* column is ***0 + 7 + 5 = 12***. Record the ***2*** in the *ones* place of the sum and regroup the ***1*** to the *tens* place.
- The sum of the digits in the *tens* column is ***8 + 8 + 1 + 1 = 18***. Record the ***8*** in the *tens* place of the sum and regroup another ***1*** to the *hundreds* place.
- The sum of the digits in the *hundreds* column is ***1 + 1 = 2***. Record it in the *hundreds* place of the sum.

We find the sum to be *282*.

Rule for Adding Decimals:

- Align the numbers in columns by place value with the decimal points directly under each other.
- Then add the digits of each column starting on the right.
- Regroup whenever the sum of a column is more than one digit.
- Place the decimal point in the sum directly under the other decimal points.

Problem 2:

Find the sum of ***67.9 + 23 + 0.34*** without a calculator.

```
 67.90
 23.00
+ 0.34
  11
 -----
 91.24
```

The decimal point in ***23*** is understood to be to the right of the *ones* place.

It is preferable to write the numbers so that all have the same number of decimal places by attaching ending zeros.

We find the sum to be *91.24*.

SUBTRACTION

Vocabulary:

In ***8 − 2 = 6***, the ***8*** is called **Minuend**, the ***2*** is the **Subtrahend**, and ***6*** is called the **Difference**.

Rule for Subtracting Whole Numbers:

- Align the two numbers in columns by place value.
- Then subtract the digits in each column starting on the right.
- Regroup whenever subtracting a larger digit from a smaller one.

Problem 3: (An alternate way of subtracting will be shown!)

Find the difference of ***8,034 − 5,679*** without a calculator.

```
 8034
−5679
 111
 ----
 2355
```

- Beginning with the *ones* column, we find that ***9*** is larger than ***4***. We change the ***4*** to ***14*** by regrouping the ***1*** to the *tens* column. ***9*** subtracted from ***14*** equals ***5*** which we record in the *ones* place of the difference.
- Continuing on to the *tens* column, we first add ***1*** to ***7*** and note that ***8*** is larger than ***3***. We change the ***3*** to ***13*** by regrouping the ***1*** to the *hundreds* column. ***8*** subtracted from ***13*** equals ***5*** which we record in the *tens* place of the difference.
- In the *hundreds* column, we first add ***1*** to ***6*** and note that ***7*** is larger than ***0***. We change the ***0*** to ***10*** by regrouping the ***1*** to the *thousands* column. ***7*** subtracted from ***10*** equals ***3*** which we record in the *hundreds* place of the difference.
- In the *thousands* column, we first add ***1*** to ***5***, and since ***5*** is smaller than ***8***, we subtract and record ***2*** in the *thousands* place of the difference.

We find the difference to be *2,355*.

Rule for Subtracting Decimals:

- Align the numbers in columns by place value with the decimal points directly under each other.
- Then subtract the digits in each column starting on the right.
- Regroup whenever subtracting a larger digit from a smaller one.
- Place the decimal point in the difference directly under the other decimal points.

Problem 4: (Again, an alternate way of subtracting will be shown!)

Find the difference of $201 - 72.35$ without a calculator.

$$\begin{array}{r} 201.00 \\ -\ 72.35 \\ \hline 128.65 \end{array}$$

The decimal point in 201 is understood to be to the right of the *ones* place.

It is preferable to write the amounts so that all have the same number of decimal places by attaching ending zeros.

We find the difference to be 128.65.

Calculator Tip: When using the calculator to find the difference of these numbers you do not have to attach zeros to the end.

MULTIPLICATION

Vocabulary:

In $8 \cdot 2 = 16$, the 8 is called **Multiplicand**, the 2 is the **Multiplier**, and 16 is called the **Product**.

NOTE:

Given that $16 = 8 \cdot 2$, we can say that 8 and 2 are **factors** of 16! That is, when a number is the product of two or more numbers, each of the latter is called a **factor** of the former.

Various Notations for Multiplication:

$8 \cdot 2 \quad 8 * 2 \quad 8 \times 2 \quad 8(2)$

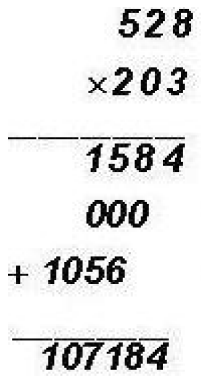
In higher mathematics parentheses ( ) are most often used to indicate multiplication!

Rule for Multiplying Whole Numbers:

- Place the two numbers one under the other.
- Multiply the multiplicand, in turn, by each digit of the multiplier starting with the ones place.
- Align the ones digit of each of these "partial products" with their multiplier digit.
- Add the "partial products" as they are aligned

Problem 5:

Find the product of ***528 × 203*** without a calculator.



Multiply ***3(528)*** and align the product (1584) under the 3 of the multiplier.
Multiply ***0(528)*** and align the product (0) under the 3 of the multiplier.
Multiply ***2(528)*** and align the product (1056) under the 3 of the multiplier.

Add the "partial products" in their current alignment assuming zeros in empty spaces.

We find the product to be *107,184*.

Problem 6:

Find the product of ***567 × 1000*** without a calculator.

A quick way to multiply a whole number by a multiple of 10 without a calculator is to move the decimal point in the multiplicand to the RIGHT as many places as there are zeros in the multiplier. Attach zeros if necessary.

There are 3 zeros in 1000, therefore, we move the decimal point in 567 three places to the RIGHT! The decimal point in a whole number is understood to be to the right of the *ones* place!

567 × 1000 = 567000

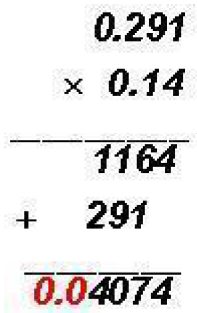
NOTE: We had to attach three zeros!

Rule for Multiplying Decimals:

- Place the two numbers one under the other.
- Ignore the decimal point and multiply just like did with whole numbers.
- Count the number of decimal places in the multiplicand and in the multiplier.
- In the product, count from the right the number of digits equal to the sum of the decimal places of the multiplicand and the multiplier. This is where the decimal point will be placed.

Problem 7:

Find the product of 0.291×0.14 without a calculator.



Three (3) decimal places plus two (2) decimal places equals five (5) decimal places.

There are NO decimal points in the "partial products."

There were not enough digits in the final product to accommodated five (5) decimal places. Therefore, we had to insert a zero to the left of the product!

We find the product to be 0.04074.

Problem 8:

Find the product of 24.5×100 without a calculator.

NOTE: A quick way to multiply a decimal by a multiple of 10 without a calculator is to move the decimal point in the mulitplicand to the RIGHT as many places as there are zeros in the multiplier. Attach zeros if necessary.

There are 2 zeros in 100, therefore, we move the decimal point of 24.5 two places to the RIGHT!

$$24.5 \times 100 = 2450$$

NOTE: We had to attach one zero!

DIVISION

Vocabulary:

In $8 \div 2 = 4$, the 8 is called **Dividend**, the 2 is the **Divisor**, and 4 is called the **Quotient**.

NOTE:

$16 \div 5$ has a quotient that consists of the whole number 3 and a **Remainder** of 1. The remainder is the part left over after long division.

Various Notations for Division:

$8 \div 2 \quad 2\overline{)8} \quad \frac{8}{2} \quad 8/2 \quad 8 / 2$

Rule:

Instead of creating a rule, the division of whole numbers and decimals will be illustrated by using examples!

Problem 9:

Find the quotient of $7\overline{)2135}$ without a calculator.

Starting on the left, find the first group of digits of the dividend that is larger or equal to the divisor. This results in the number ***21***.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \end{array}$$

Divide ***21*** by ***7*** and write ***3*** above the rightmost digit of ***21***.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ 21 \end{array}$$

Multiply ***3*** by ***7*** and write this product below the digits ***21*** of the dividend. Align places!

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ -\,21 \\ \hline 0 \end{array}$$

Subtract the product from ***21*** and write the difference ***0*** below the product aligning places.

$$\begin{array}{r} 3 \\ 7\overline{)2135} \\ -\,21 \\ \hline 03 \end{array}$$

To the right of the difference ***0*** write the next digit of the dividend, which is ***3***.

$$\begin{array}{r} 30 \\ 7\overline{)2135} \\ -\,21 \\ \hline 03 \\ -\,0 \\ \hline 3 \end{array}$$

Please note that ***7*** divides into ***3*** zero times! That's why the next digit of the quotient is ***0***.

$$\begin{array}{r} 30 \\ 7\overline{)2135} \\ -\,21 \\ \hline 03 \\ -\,0 \\ \hline 35 \end{array}$$

To the right of the difference we write the next digit of the dividend, which is ***5***.

Finally,

$$
\begin{array}{r}
305 \\
7\overline{)\,2135} \\
-\,21 \\
\hline
03 \\
-\,0 \\
\hline
35 \\
-\,35 \\
\hline
0
\end{array}
$$

We find the quotient to be *305*. There is NO remainder.

Problem 10:

Find the quotient of $12\overline{)340}$ without a calculator.

$$
\begin{array}{r}
28 \\
12\overline{)340} \\
-\,24 \\
\hline
100 \\
-\,96 \\
\hline
4
\end{array}
$$

We find the quotient to be *28*. There is a remainder of *4*.

Problem 11:

Find the quotient of $8\overline{)5.6}$ without a calculator.

Since the dividend is a decimal number, our first task is to insert a decimal point into the quotient. It must be placed right above the decimal point of the dividend.

$$
\begin{array}{r}
. \\
8\overline{)5.6}
\end{array}
$$

Since ***8*** is larger than the whole number ***5*** of the dividend, the whole number part of the quotient will be ***0***.

$$\begin{array}{r} 0. \\ 8\overline{)5.6} \end{array}$$

To continue the division process, we now have to use a digit from the fractional part of the dividend. We will ignore the decimal point in the dividend and divide 8 into 56. However, all digits of the quotient MUST now be placed to the right of the decimal point.

$$\begin{array}{r} 0.7 \\ 8\overline{)\ 5.6} \\ -\ 5\ 6 \\ \hline 0 \end{array}$$

We find the quotient to be 0.7. There is NO remainder.

Problem 12:

Find the quotient of $2.3\overline{)3.68}$ without a calculator.

NOTE: The divisor must always be changed to a whole number!

Since the divisor is NOT a whole number, the first thing we will do is to move its decimal point so that it is on the right side of all digits.

Next, we'll move the decimal point in the dividend to the right as many places as the decimal point was moved in the divisor. Attach ending zeros if necessary!

$$23\overline{)36.8}$$

Since the divisor is now a whole number, we'll insert a decimal point into the quotient. Remember that it must be placed right above the decimal point of the dividend.

$$\begin{array}{r} . \\ 23\overline{)36.8} \end{array}$$

Since 23 is larger than the whole number 36 of the dividend and therefore divides into it (once), the whole number part of the quotient will be 1.

$$\begin{array}{r} 1. \\ 23\overline{)36.8} \\ -\ 23 \\ \hline 13 \end{array}$$

To continue the division process, we now have to use a digit from the fractional part of the dividend. We will ignore the decimal point in the dividend and divide ***23*** into ***138***. However, all digits of the quotient MUST now be placed to the right of the decimal point.

```
      1.6
23 ) 36.8
   – 23
     ——
     13 8
   – 13 8
     ————
        0
```

We find the quotient to be *1.6*. There is NO remainder.

Problem 13:

Find the quotient of ***567 ÷ 10000*** without a calculator.

NOTE: A quick way to divide a number by a multiple of 10 without a calculator is to move the decimal point in the dividend to the LEFT as many places as there are zeros in the divisor. Insert zeros if necessary.

There are 4 zeros in 10000, therefore, we move the decimal point in 567 four places to the LEFT! The decimal point in a whole number is understood to be to the right of the *ones* place!

567 ÷ 10000 = 0.0567

Here we had to insert one zero to the right of the decimal point.

Problem 14:

Find the quotient of ***786580 ÷ 100*** without a calculator.

Here we move the decimal point two places to the left!

786580 ÷ 100 = 7865.8

Problem 15:

Find the quotient of ***198.78 ÷ 10***.

Here we move the decimal point one place to the left!

198.78 ÷ 10 = 19.878