DETAILED SOLUTIONS AND CONCEPTS - SIGNED NUMBERS
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PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

Many physical phenomena have values that are less than zero. For example, when a temperature is colder than zero degrees, how is this expressed numerically? When a withdrawal on a bank account exceeds the amount of money in the account, how is this expressed numerically?

Negative numbers are needed to express these values. Then the numbers can be expanded to include positive and negative numbers. These so-called **signed numbers** can be formed from all types of numbers such as natural numbers, fractions, decimals, mixed numbers, etc.
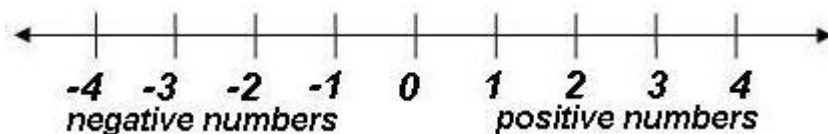
Integers

The *Integers* include all of the *Natural Numbers* and their negatives and the number *0*.

{... , - 4, -3, -2, -1, 0, 1, 2, 3, 4, ...}

The negative numbers are pronounced "negative 1", "negative 2", "negative 3", etc. The negative sign tells us that the numbers are to the left of zero and is referred to as a **directional sign**.

DO NOT SAY "minus 1", "minus 2", "minus 3", etc.

Integers can be arranged on a number line to show a visual representation of the relationship of their size.

As you can see from the number line, it is not necessary, nor is it customary and standard, to include the positive sign for positive numbers. You sometimes may want to include it to draw attention to it or to emphasize the positive nature of the number. However, negative signs must always be included.

Positive numbers are larger than negative numbers. The further left of **0** a number is located, the smaller it is.

For example,

-1 > -10 but **10 > 1**

-1 < 0 but **0 < 1**

Rational Numbers

Any type of number that can be written as the quotient of two *Integers*. This includes all terminating and repeating decimals, fractions, and the *Integers*.

Problem 1:

Which of the following numbers is the smallest?

0 , 1, $-\dfrac{1}{3}$ **, -3**

Problem 2:

Which of the following numbers is the smallest?
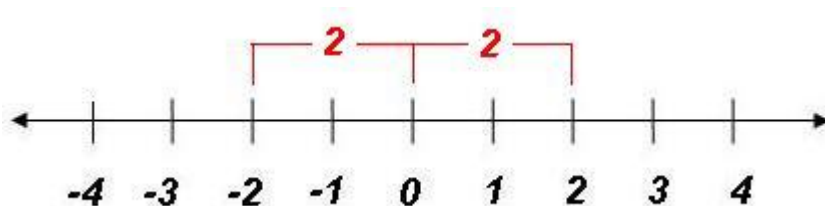
-10, -20, -5.3, 0

Problem 3:

Which of the following numbers is the largest?

-1, 0.01, 0.1, -0.2

Absolute Value

The *Absolute Value* expresses the distance between a number and **0**. Let's look at the number line one more time.

We can see that the distance between **0** and **2** is **2**. BUT, the distance between **0** and **-2** is also **2**. We can express this as follows:

$|2| = 2$ means that the distance between **0** and **2** is **2**. It is pronounced "the absolute value of two equals 2."

$|-2| = 2$ means that the distance between **0** and **-2** is also **2**. It is pronounced "the absolute value of negative 2 equals 2."

Problem 4:

Evaluate $|-7|$.

This means that the distance between **0** and **-7** is **7**. Therefore, $|-7| = 7$.

Problem 5:

Evaluate $|5|$.

This means that the distance between **0** and **5** is **5**. Therefore, $|5| = 5$.

Problem 6:

Evaluate $|0|$.

This means that the distance between **0** and **0** is **0**. Therefore, $|0| = 0$.

Problem 7:

Evaluate $\left|-\frac{2}{3}\right|$.

This means that the distance between **0** and $-\frac{2}{3}$ is $\frac{2}{3}$. Therefore, $\left|-\frac{2}{3}\right| = \frac{2}{3}$.

Problem 8:

Evaluate $\left|\frac{7}{8}\right|$.

This means that the distance between **0** and $\frac{7}{8}$ is $\frac{7}{8}$. Therefore, $\left|\frac{7}{8}\right| = \frac{7}{8}$.

Equivalent Signed Fractions

A fraction has three signs:

- the sign of the fraction placed at the same level as the fraction bar
- the sign of the numerator
- the sign of the denominator

Problem 9:

Change $\dfrac{-5}{6}$ to two different equivalent fractions.

The fraction $\dfrac{-5}{6}$ can be expressed as $\dfrac{5}{-6}$ and $-\dfrac{5}{6}$.

Note that positive signs do not have to be written and are usually left off!

Problem 10:

Change $-\dfrac{3}{4}$ to two different equivalent fractions.

The fraction $-\dfrac{3}{4}$ can be expressed as $\dfrac{-3}{4}$ and $\dfrac{3}{-4}$.

Note that positive signs do not have to be written and are usually left off!

Addition of Signed Numbers

Rule:

- If numbers have the same sign, add their absolute value. The sum has the same sign as the numbers.
- If numbers have different signs, subtract the smaller absolute value from the larger absolute value. Give the sum the sign of the number with the larger absolute value.

Problem 11:

Find the sum of **-2 + (-9)**.

Note: To separate the directional sign of a number from the operational sign, it is customary to enclose the signed number in parentheses!

We know that $|-2| = 2$ and $|-9| = 9$. We find the sum of 2 and 9 and then give it a negative sign because the numbers are negative.

-2 + (-9) = -11

Hint: If you look at the negative sign as a gambling loss, you can say that "a loss plus another loss equals a greater loss."

Problem 12:

Find the sum of **+2 + (+9)**.

Note: It is not necessary nor is it customary and standard to include the positive sign for positive numbers. It is only done here to illustrate the concept of addition of signed numbers.

Most commonly, you will see **2 + 9**.

Again $|2| = 2$ and $|9| = 9$. We find the sum of 2 and 9 and then give it a positive sign because the numbers are positive.

+2 + (+9) = 11

Hint: If you look at the positive sign as gambling winnings, you can say that "winnings plus more winnings equals greater winnings."

Problem 13:

Find the sum of **-2 + (+9)**.

We know that $|-2| = 2$ and $|9| = 9$. We find the difference of 2 and 9 and then give it a positive sign because the number with the larger absolute value is positive.

-2 + (+9) = +7 or **7**

Find the sum of **2 + (-9)**.

Again $|2| = 2$ and $|9| = 9$. We find the difference of 2 and 9 and then give it a negative sign because the number with the larger absolute value is negative.

2 + (-9) = -7

Subtraction of Signed Numbers

Rule:

- Change the operational sign from minus to plus.
- Change the directional sign of the subtrahend to the opposite sign.
- Apply the appropriate rule for adding signed numbers.

Problem 15:

Carry out the following subtractions. Notice that the operational signs change from minus to plus and the directional sign of the subtrahend changes to the opposite sign.

a. **12 - (+3) = +12 + (-3) = 9**

b. **5 - (+7) = +5 + (-7) = -2**

c. **-3 - (-5) = -3 + (+ 5) = 2**

d. **0 - (-5) = 0 + (+ 5) = 5**

e. **0 - (+5) = 0 + (- 5) = -5**

Omitting Signs under Addition and Subtraction

Writing mathematical expressions that include operational and directional signs is cumbersome and can be confusing. In general, we omit one of the signs as often as possible. When two signs are written between two numbers, we can simplify this as follows:

double sign   **single sign**

+(+)   **+**

-(-)   **+**

+(-)   **-**

-(+)   **-**

For example,

instead of **+3 + (+5)**, we write **3 + 5** and "think" 3 **plus** a positive 5;

instead of **+3 - (-5)**, we write **3 + 5** and "think" 3 **plus** a positive 5;

instead of +3 + **(-5)**, we write **3 - 5** and "think" 3 **plus** a negative 5;

instead of **+3 -(+5)**, we write **3 - 5** and "think" 3 **plus** a negative 5.

Problem 16:

Find the sum of $\dfrac{-3}{4} + \dfrac{-4}{7}$ and write as a mixed number.

When working with fractions it is especially important to combine double signs to a single sign first. Before we carry out the addition above, we'll change the signs as follows:

$$\frac{-3}{4} - \frac{4}{7}$$

Then, using the LCD **28**, we find

$$\frac{-3(7)}{4(7)} - \frac{4(4)}{7(4)} = \frac{-21}{28} - \frac{16}{28}$$
$$= \frac{-21-16}{28}$$
$$= \frac{-37}{28}$$
$$= -1\frac{9}{28}$$

Problem 17:

Find the difference of $\dfrac{-3}{7} - \dfrac{4}{-7}$.

Again, we first change the double sign to a single sign!

$$\dfrac{-3}{7} + \dfrac{4}{7}$$

and then, since the denominators are equal, we carry out the operation

$$\dfrac{-3}{7} + \dfrac{4}{7} = \dfrac{-3+4}{7} = \dfrac{1}{7}$$

Problem 18:

Evaluate **3 + 5 - 6**.

Using the *Order of Operation*, we'll combine the first two numbers

8 - 6

and then carry out the last two numbers to get **2**.

Problem 19:

Evaluate **-18 + 12 - 7**.

Using the *Order of Operation*, we'll combine the first two numbers

-6 - 7

and then combine the last two numbers to get **-13**.

Problem 20:

Evaluate **6 - 3 - (-10) - 7 + (-13) + 33**.

First, we will rewrite the expression by combining the operational and directional signs.

6 - 3 + 10 - 7 -13 + 33

Using the *Order of Operation*, we'll combine the first two numbers

3 + 10 - 7 - 13 + 33

combine the next two numbers . . .

13 - 7 - 13 + 33

combine the next two numbers . . .

6 - 13 + 33

combine the next two numbers . . .

- 7 + 33

and then combine the last two numbers to get **26**.

Multiplication of Signed Numbers

Rule:

- Multiply the absolute value of the numbers.
- The product of two numbers having the same directional sign is positive.
- The product of two number having different directional signs is negative.

Problem 21:

Find the product of *-2(-9)*.

We know that $|-2| = 2$ and $|-9| = 9$. We find the product of 2 and 9 and then give it a positive sign because both signed numbers have the same sign.

-2(-9) = +18 or *18* When a number is positive the + sign is usually omitted!

Problem 22:

Find the product of *2(9)*.

We know that $|2| = 2$ and $|9| = 9$. We find the product of 2 and 9 and then give it a positive sign because both signed numbers have the same sign.

2(9) = +18 or *18*

Problem 23:

Find the product of *-2(9)*.

We know that $|-2| = 2$ and $|9| = 9$. We find the product of 2 and 9 and then give it a negative sign because the signed numbers have different signs.

-2 (9) = -18

Problem 24:

Find the product of **2(-9)**.

We know that $|2| = 2$ and $|-9| = 9$. We find the product of 2 and 9 and then give it a negative sign because the signed numbers have different signs.

2(-9) = -18

Problem 25:

Find the product of **0(-9)**.

We know that when **0** is a factor of a product the product equals **0**.

0 (-9) = 0

Problem 26:

Find the product of **4(-3)(-2)(6)(-5)**.

Using the *Order of Operation*, we'll multiply the first two numbers

-12(-2)(6)(-5)

Again, we multiply the first two numbers to get **24(6)(-5)**

and again ... **144(-5)**

then we multiply the final two numbers to get **-720**.

Problem 27:

Find $\left(-\dfrac{1}{3}\right)^2$.

We know that $\left(-\dfrac{1}{3}\right)^2 = -\dfrac{1}{3}\left(-\dfrac{1}{3}\right)$ or $\dfrac{-1}{3}\left(\dfrac{-1}{3}\right)$

and since a negative number multiplied by a negative number results in a positive product, we can say

$$\left(-\dfrac{1}{3}\right)^2 = \dfrac{-1(-1)}{3(3)}$$

$$= \dfrac{1}{9}$$

Problem 28:

Find the product of **3(-2)(-1)(0)(-9)**.

We know that when **0** is a factor of a product the product equals **0**.

(-2)(-1)(0)(-9) = 0

Problem 29:

Find the product of $\dfrac{-5}{6}\left(\dfrac{7}{-9}\right)$.

$$\frac{-5}{6}\left(\frac{7}{-9}\right) = \frac{-5(7)}{6(-9)}$$

$$= \frac{-35}{-54}$$

$$= \frac{35}{54}$$

Division of Signed Numbers

Rule:

- Divide the absolute value of the numbers.
- The quotient of two numbers having the same directional sign is positive.
- The quotient of two numbers having different directional signs is negative.

Problem 30:

Find the quotient of $\dfrac{-9}{-3}$.

We know that $|-3| = 3$ and $|-9| = 9$. We find the quotient of 3 and 9 and then give it a positive sign because both signed numbers have the same sign.

$$\frac{-9}{-3} = +3 \text{ or } 3$$

Problem 31:

Find the quotient of $\dfrac{9}{3}$.

We know that $|3| = 3$ and $|9| = 9$. We find the quotient of 3 and 9 and then give it a positive sign because both signed numbers have the same sign.

$$\frac{9}{3} = +3 \text{ or } 3$$

Problem 32:

Find the quotient of $\dfrac{9}{-3}$.

We know that $|-3| = 3$ and $|9| = 9$. We find the quotient of 3 and 9 and then give it a negative sign because the signed numbers have different signs.

$$\frac{9}{-3} = -3$$

Problem 33:

Find the quotient of $\dfrac{-9}{3}$.

We know that $|3| = 3$ and $|-9| = 9$. We find the quotient of 3 and 9 then give it a negative sign because the signed numbers have different signs.

$$\frac{-9}{3} = -3$$

Problem 34:

Find the quotient of $\dfrac{-9}{0}$.

The denominator of any fraction cannot have the value **0**. If the denominator of a fraction is **0**, we say that the value of this fraction is **undefined**.

Find the quotient of $\dfrac{0}{-9}$.

A numerator is allowed to take on the value of **0** in a fraction. Any fraction (denominator not equal to **0**) with a numerator equal to **0** has an overall value of **0**.

$$\frac{0}{-9} = 0$$

Problem 36:

Find the quotient of $\dfrac{-4}{3} \div \dfrac{5}{-7}$.

First, we'll change this division to a multiplication:

$$\frac{-4}{3} \cdot \frac{-7}{5}$$

and then we'll carry out the multiplication

$$\frac{-4}{3} \cdot \frac{-7}{5} = \frac{-4(-7)}{3(5)}$$
$$= \frac{28}{15}$$
$$= 1\frac{13}{15}$$

Expressing Signed Mixed Numbers as Improper Fractions

In arithmetic we learned that $3\dfrac{1}{2}$ means $3 + \dfrac{1}{2}$!!!!!!!!!!!!!!!!!!!

Then, when we change a mixed number to an improper fraction, we multiply the whole number by the denominator of the fractional part and to this product we add the numerator of the fractional part. The denominator stays the same throughout.

That is, $3\dfrac{1}{2} = \dfrac{3 \cdot 2 + 1}{2} = \dfrac{6 + 1}{2} = \dfrac{7}{2}$

Let's convert $-3\frac{1}{2}$ to an improper fraction.

NOW THE FOLLOWING IS REALLY IMPORTANT ....

$-3\frac{1}{2}$ MEANS $-3-\frac{1}{2}$!!!!!!!!!!!!!!!!!!!!

Therefore, we convert as follows:

$$-3\frac{1}{2} = \frac{-3\cdot 2 - 1}{2} = \frac{-6-1}{2} = \frac{-7}{2} = -\frac{7}{2}$$

PLEASE NOTE THAT $-3\frac{1}{2}$ **DOES NOT MEAN** $-3+\frac{1}{2}$ **!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!**

Problem 37:

Convert $-4\frac{7}{8}$ to an improper fraction.

$$-4\frac{7}{8} = \frac{-4(8)-7}{8} = \frac{-39}{8}$$

Problem 38:

Convert $-1\frac{4}{9}$ to an improper fraction.

$$-1\frac{4}{9} = \frac{-1(9)-4}{9} = \frac{-13}{9}$$

Problem 39:

Find the sum of $-6\frac{1}{2}+\left(-2\frac{2}{3}\right)$ and write as a mixed number.

We first change the double sign to a single sign!

$$-6\frac{1}{2} - 2\frac{2}{3}$$

Here we are dealing with signed mixed numbers. We will change them to improper fractions.

Please remember that $-6\dfrac{1}{2} = -6 - \dfrac{1}{2}$ and $-2\dfrac{2}{3} = -2 - \dfrac{2}{3}$ **!!!**

Therefore, we get

$$-6\dfrac{1}{2} - 2\dfrac{2}{3} = -\dfrac{13}{2} - \dfrac{8}{3}$$

Both fractions have a different denominator. Before we can simplify, we will have to make the denominators the same. We do this by using the least common denominator (LCD). This is the smallest number that is evenly divisible (no remainder) by **2** and **3**.

2 and **3** are prime numbers. Therefore, the least common denominator (LCD) is found by calculating their product, which is **6**.

$$\begin{aligned}
-6\dfrac{1}{2} - 2\dfrac{2}{3} &= -\dfrac{13}{2}\cdot\dfrac{3}{3} - \dfrac{8}{3}\cdot\dfrac{2}{2} \\
&= -\dfrac{39}{6} - \dfrac{16}{6} \\
&= -\dfrac{55}{6} \\
&= -9\dfrac{1}{6}
\end{aligned}$$

The **Order of Operation** is a set of rules that allow us to interpret a mathematical expression, such as **2+3(5)**, for example. It determines which mathematical operations should be performed in what order.

1. First, carry out operations in numerators and denominators of fractions and within radicals. You might have to use steps 2 through 5!

2. Then carry out operations enclosed by **grouping symbols**, such as parenthesis ( ), brackets [ ], and braces { }.

    NOTE: Given grouping symbols, the value enclosed by the innermost symbol is done first. The innermost symbol is usually a set of parentheses.

3. Exponential expressions are evaluated next.

4. Then we divide and multiply, whichever comes first, in order from left to right.

5. Lastly we add and subtract, whichever comes first, in order from left to right.

Problem 40:

Use the **Order of Operation** to simplify $-6 \div (5 - 7) - 4(-6 + 1)^2 - 5^2$.

We'll do the subtractions within parentheses first.

$-6 \div (-2) - 4(-5)^2 - 5^2$

Next, we'll raise the appropriate numbers to the second power.

$-6 \div (-2) - 4(25) - 25$

> Please note that $(-5)^2 = 25$, but $-5^2 = -1(5^2) = -25$. By the *Order of Operation*, exponential expressions are simplified **BEFORE** we multiply (in this case by *-1*)!

Next, we'll divide and multiply from left to right. In this case, we can do both operations at the same time, but only because they are separated by a minus sign.

$3 - 100 - 25$

and finally, we subtract from left to right to get *-122*.

Problem 41:

Use the **Order of Operation** to simplify $7(3 + 9) - (6 - 3)$.

We'll do the addition and subtraction within parentheses first.

$7(12) - 3$

Next, we'll multiply.

$84 - 3$

and finally, we subtract to get *81*.

Problem 42:

Use the **Order of Operation** to simplify $2(6 - 8)^2 - (4 - 5)^2$.

We'll do the subtractions within parentheses first.

$2(-2)^2 - (-1)^2$

Next, we'll take care of the exponents.

2(4) - 1

then we'll multiply

8 - 1

and finally, we subtract to get *7*.

Problem 43:

Use the **Order of Operation** to simplify $25 - 25 \div (-7 + 2)$.

We need to take care of the parentheses first.

25 - 25 ÷ (-5)

Then, we'll have to divide.

25 + 5

Finally, we'll add to get *30*.

Problem 44:

Use the **Order of Operation** to simplify $(-10 + 3)^2 + 4(-2) - (10 + 5) \div (-5)$.

We need to take care of the parentheses first.

$(-7)^2 + 4(-2) - 15 \div (-5)$

Next, we'll need to evaluate the exponential expression.

49 + 4(-2) - 15 ÷ (-5)

Then, we'll have to multiply and divide in order from left to right. In this case, we can do both operations at the same time, but only because they are separated by a minus sign.

49 - 8 + 3

Finally, we'll subtract and and add in order from left to right to get *44*.

Problem 45:

Use the **Order of Operation** to simplify $-6^2 + 3(-18) \div (-5 - 4)$.

We'll do the subtraction within parentheses first.

$-6^2 + 3(-18) \div (-9)$

Next, we'll evaluate the exponent.

-36 + 3(-18) ÷ (-9)

Please note that **(-6)^2** is NOT EQUAL to **-6^2**.

(-6)^2 = -6(-6) = 36 BUT **-6^2 = -1(6)(6) = -36**

Now, we have to multiply and divide IN ORDER from left to right. We'll do the multiplication first!

-36 - 54 ÷ (-9)

Then, we'll do the division.

-36 + 6

Finally, we'll subtract to get **-30**.