



**DETAILED SOLUTIONS AND CONCEPTS**  
**GRAPHICAL REPRESENTATION OF LINEAR INEQUALITIES IN TWO VARIABLES**  
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Please Send Questions and Comments to [ingrid.stewart@csn.edu](mailto:ingrid.stewart@csn.edu). Thank you!

**PLEASE NOTE THAT YOU CANNOT USE A CALCULATOR ON THE ACCUPLACER - ELEMENTARY ALGEBRA TEST! YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!**

### **Linear Inequalities in Two Variables**

An inequality in which two letters are used for variables and the exponent of the variables is **1**. Following are a few examples of linear inequalities in two variables:

$$2x - y \geq 4 \quad y \leq -3x - 6 \quad y < 2x \quad 5x - 3y > 7$$

$y \geq 0$      $x \leq 0$     These inequalities look different from the previous four examples. There is only one variable in each one! See below!

### **Strategy for Graphing Linear Inequalities in Two Variables**

- Replace the inequality sign with an equal sign and graph this linear equation. This is our "boundary" line. Make a solid line if the boundary line is included ( $\leq$  or  $\geq$ ) or a dashed line if the boundary line is not included ( $<$  or  $>$ ).
- Pick a test point that is not on the boundary line and replace the x- and y-variable with it.

If the test point makes a true statement with the original inequality, shade the side containing the test point.

If the test point makes a false statement with the original inequality, shade the side opposite the side containing the test point.

### Problem 1:

Graph the linear inequality  $2x - y \geq 4$ .

1. First we will graph the equality  $2x - y = 4$ .

Coordinates of the x-intercept (when  $y = 0$ ):

$$2x - 0 = 4$$

$$x = 2$$

The coordinates of the x-intercept are  $(2, 0)$ .

Coordinates of the y-intercept (when  $x = 0$ ):

$$2(0) - y = 4$$

$$y = -4$$

The coordinates of the y-intercept are  $(0, -4)$ .

Coordinates of one additional point using the *Point-by-Point Plotting Method*:

Let's choose  $x = 4$ . Please note that we could have chosen any other value, but looking at the existing points, it seemed that 4 was a reasonable choice.

Next, we will replace the  $x$  in  $2x - y = 4$  with 4.

That is,  $2(4) - y = 4$

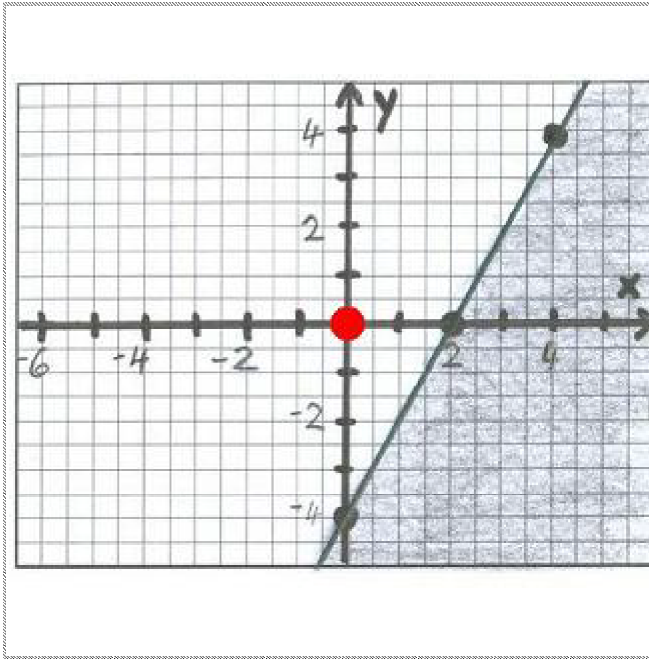
$$8 - y = 4$$

and  $y = 4$

The coordinates of the additional point are  $(4, 4)$ .

Since we have a "greater than or equal"  $\geq$  inequality, we draw a **solid line**, which is our boundary line.

2. Next we are going to pick a test point. The easiest one is  $(0, 0)$  since it is not on the boundary line.



Using the original inequality  $2x - y \geq 4$  and  $(0, 0)$ , we will write the following statement:

$$2(0) - 0 \geq 4$$

and  $0 \geq 4$ . This is a false statement since 0 is not greater than or equal to 4.

Therefore, we shaded the side opposite the side containing the test point.

**The boundary line together with the shaded area represents the graph of the inequality. Sometimes, this is also referred to as the "solution" of the inequality in two variables!**

### Problem 2:

Graph the linear inequality  $y \leq -3x - 6$ .

1. First we will graph the equality  $y = -3x - 6$ .

Coordinates of the x-intercept (when  $y = 0$ ):

$$0 = -3x - 6$$

$$3x = -6$$

$$x = -2$$

The coordinates of the x-intercept are  $(-2, 0)$ .

Coordinates of the y-intercept (when  $x = 0$ ):

$$y = -3(0) - 6 = -6$$

The coordinates of the y-intercept are  $(0, -6)$ .

Coordinates of one additional point using the *Point-by-Point Plotting Method*:

Let's choose  $x = -4$ . Please note that we could have chosen any other value, but looking at the existing points, it seemed that -4 was a reasonable choice.

Next, we will replace the  $x$  in  $y = -3x - 6$  with  $-4$ .

That is,  $y = -3(-4) - 6$

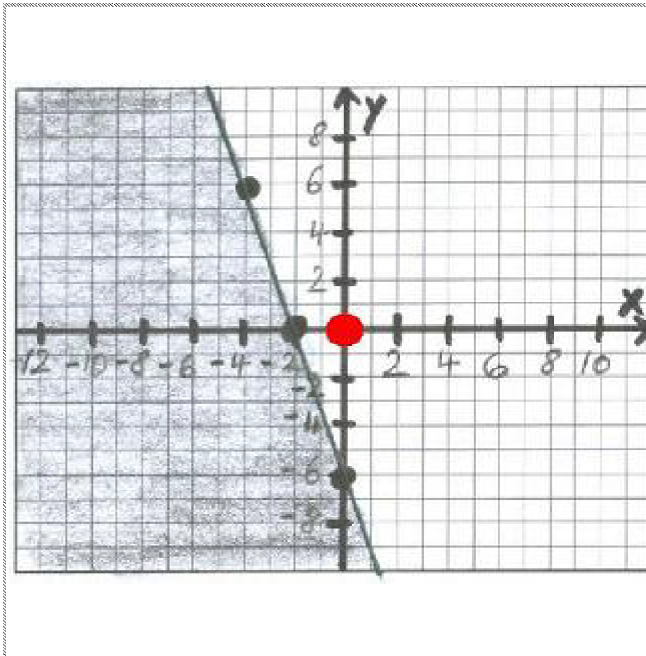
$$y = 12 - 6$$

$$\text{and } y = 6$$

The coordinates of the additional point are  $(-4, 6)$ .

Since we have a "less than or equal"  $\leq$  inequality, we draw a **solid line**, which is our boundary line.

2. Next we are going to pick a test point. The easiest one is  $(0, 0)$  since it is not on the boundary line.



Using the original inequality  $y \leq -3x - 6$  and  $(0, 0)$ , we will write the following statement:

$$0 \leq -3(0) - 6$$

and  $0 \leq -6$ . This is a false statement since 0 is not less than or equal to -6.

Therefore, we shaded the side opposite the side containing the test point.

**The boundary line together with the shaded area represents the graph of the inequality. Sometimes, this is also referred to as the "solution" of the inequality in two variables!**

### Problem 3:

Graph the linear inequality  $y > 2x$ .

1. First we will graph the equality  $y = 2x$ .

Coordinates of the x-intercept (when  $y = 0$ ):

$$0 = 2x$$

$$x = 0$$

The coordinates of the x-intercept are  $(0, 0)$ .

Coordinates of the y-intercept (when  $x = 0$ ):

$$y = 2(0)$$

$$y = 0$$

The coordinates of the y-intercept are  $(0, 0)$ .

We find that the coordinates of the x- and y-intercept are the same. That is, we didn't get two data points ... and without two data points we cannot graph our linear equation. Here we definitely have to use the *Point-by-Point Plotting Method* to find the coordinates of some other points.

Let's choose a positive and a negative value for  $x$ . Not too close to the point  $(0,0)$  so that we can draw an accurate line.

Let's use  $x = 4$ ,

then  $y = 2(4)$

and  $y = 8$

The coordinates of the additional point are  $(4, 8)$ .

Next, let's use  $x = -4$ ,

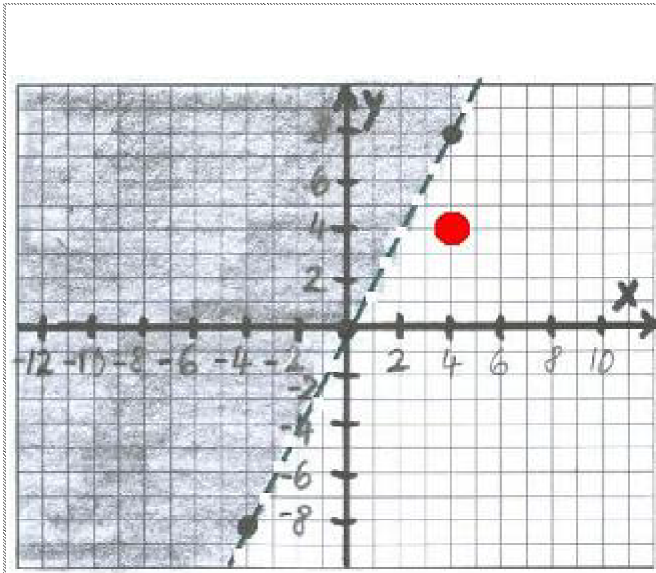
then  $y = 2(-4)$

and  $y = -8$

The coordinates of the second point are  $(-4, -8)$ .

Since we have strictly a "less than"  $<$  inequality, we draw a **dashed line**, which is our boundary line.

2. Next we are going to pick a test point. Please note that the test point  $(0, 0)$  is on the boundary line, therefore, we must use a different test point. Any point will do, as long as it is not on the boundary line. Let's use  $(4, 4)$ .



Using the original inequality  $y > 2x$  and  $(4, 4)$ , we will write the following statement:

$$4 > 2(4)$$

and  $4 > 8$ . This is a false statement since 4 is not greater than 8.

Therefore, we shaded the side opposite the side containing the test point.

**The boundary line together with the shaded area represents the graph of the inequality. Sometimes, this is also referred to as the "solution" of the inequality in two variables!**

#### Problem 4:

Graph the linear equation  $3y > -9x$ .

1. First we will graph the equality  $3y = -9x$ .

Coordinates of the x-intercept (when  $y = 0$ ):

$$3(0) = -9x$$

$$x = 0$$

The coordinates of the x-intercept are  $(0, 0)$ .

Coordinates of the y-intercept (when  $x = 0$ ):

$$3y = -9(0)$$

$$0 = 0$$

The coordinates of the y-intercept are  $(0, 0)$ .

We find that the coordinates of the x- and y-intercept are the same. That is, we didn't get two data points ... and without two data points we cannot graph our linear equation. Here we definitely have to use the *Point-by-Point Plotting Method* to find the coordinates of some other points.

Let's choose a positive and a negative value for  $x$ . Not too close to the point  $(0,0)$  so that we can draw an accurate line.

Let's use  $x = 2$ ,

then  $3y = -9(2)$

and  $y = -6$

The coordinates of the additional point are  $(2, -6)$ .

Next, let's use  $x = -2$ ,

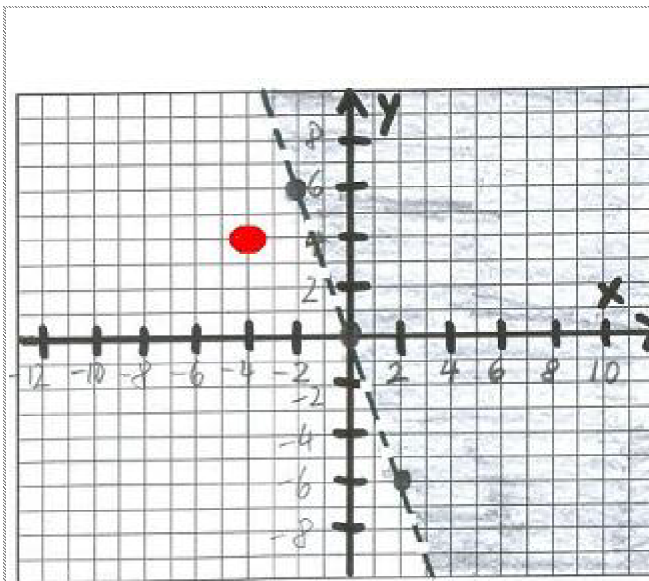
then  $3y = -9(-2)$

and  $y = 6$

The coordinates of the second point are  $(-2, 6)$ .

Since we have strictly a "greater than"  $>$  inequality, we draw a **dashed line**, which is our boundary line.

2. Next we are going to pick a test point. Please note that the test point  $(0, 0)$  is on the boundary line, therefore, we must use a different test point. Any point will do, as long as it is not on the boundary line. Let's use  $(-4, 4)$ .



Using the original inequality  $3y > -9x$  and  $(-4, 4)$ , we will write the following statement:

$$3(4) > -9(-4)$$

and  $12 > 36$ . This is a false statement since 12 is not greater than 36.

Therefore, we shaded the side opposite the side containing the test point.

**The boundary line together with the shaded area represents the graph of the inequality. Sometimes, this is also referred to as the "solution" of the inequality in two variables!**

### Problem 5:

Graph the linear inequality  $y \geq 0$ .

This inequality looks different from the previous four examples. There is only one variable in it! So how can we call it a "linear inequality in two variables" ?????? Oddly enough, it still fall into this category with the "missing" variable being allowed to take on any value.

**Recall the following:**

#### Equation of Horizontal Lines

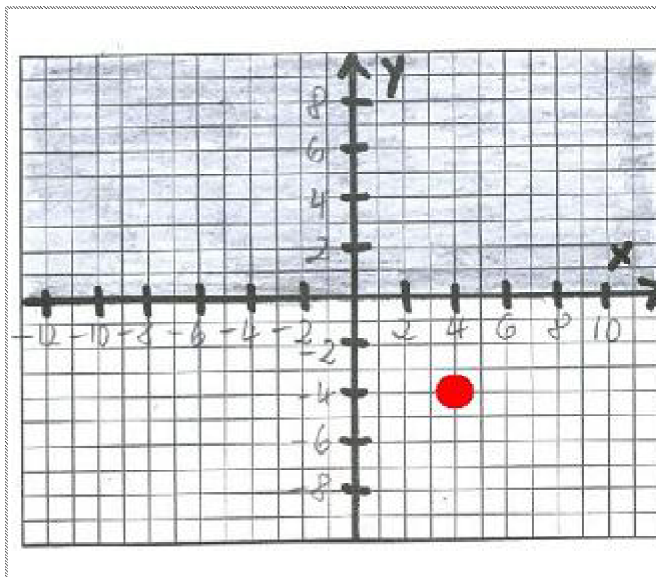
The equation of a **horizontal line** is  $y = b$ , where  $b$  is a constant and the y-coordinate of any point on the line.

Horizontal lines do not have an x-intercept, but they do have a y-intercept at  $(0, b)$ .

We know that the graph of  $y = 0$  is a horizontal line, **specifically, it is the equation of the x-axis.**

Since we have a "greater than or equal"  $\geq$  inequality the x-axis is included!

Next we are going to pick a test point. Since the point  $(0, 0)$  is on the boundary line, we must use a different test point. Any point will do, as long as it is not on the boundary line. Let's use  $(4, -4)$ .



Using the original inequality  $y \geq 0$  and  $(4, -4)$ , we will write the following statement:

$-4 \geq 0$ . This is a false statement since  $-4$  is not greater than  $0$ .

Therefore, we shaded the side opposite the side containing the test point.

**The boundary line together with the shaded area represents the graph of the inequality. Sometimes, this is also referred to as the "solution" of the inequality in two variables!**

## Problem 6:

Graph the linear inequality  $x \leq 0$ .

This inequality also looks different from Examples 1- 4.

**Recall the following:**

### Equation of Vertical Lines

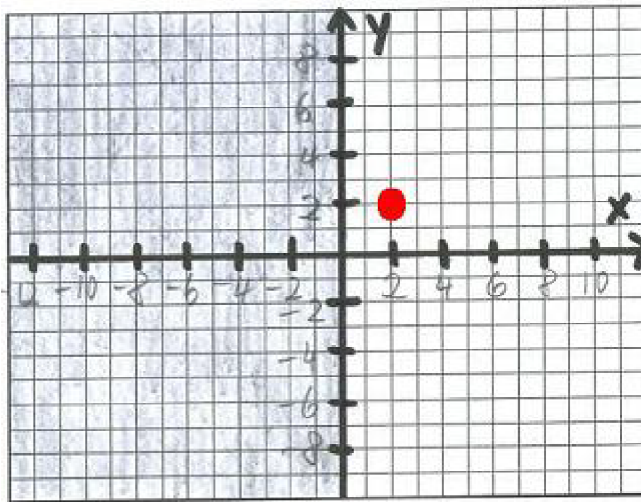
The equation of a **vertical line** is  $x = a$ , where  $a$  is a constant and the x-coordinate of any point on the line.

Vertical lines do not have a y-intercept, but they do have an x-intercept at  $(a, 0)$ .

We know that the graph of  $x = 0$  is a vertical line, **specifically, it is the equation of the y-axis.**

Since we have a "less than or equal"  $\leq$  inequality the y-axis is included!

Next we are going to pick a test point. Since the point  $(0, 0)$  is on the boundary line, we must use a different test point. Any point will do, as long as it is not on the boundary line. Let's use  $(2, 2)$ .



Using the original inequality  $x \leq 0$  and  $(2, 2)$ , we will write the following statement:

$2 \leq 0$ . This is a false statement since 2 is not less than 0.

Therefore, we shaded the side opposite the side containing the test point.

**The boundary line together with the shaded area represents the graph of the inequality. Sometimes, this is also referred to as the "solution" of the inequality in two variables!**

### Problem 7:

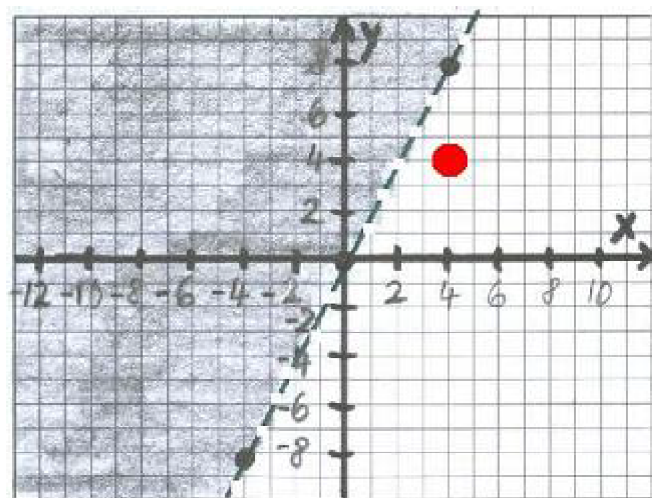
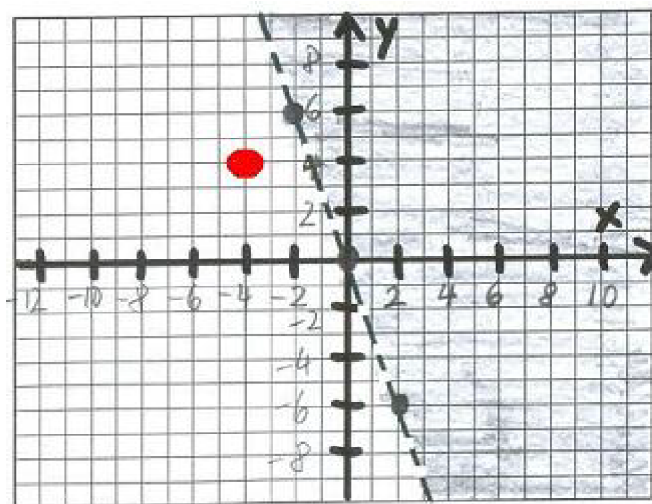
Solve the following system of linear inequalities:

$$3y > -9x$$

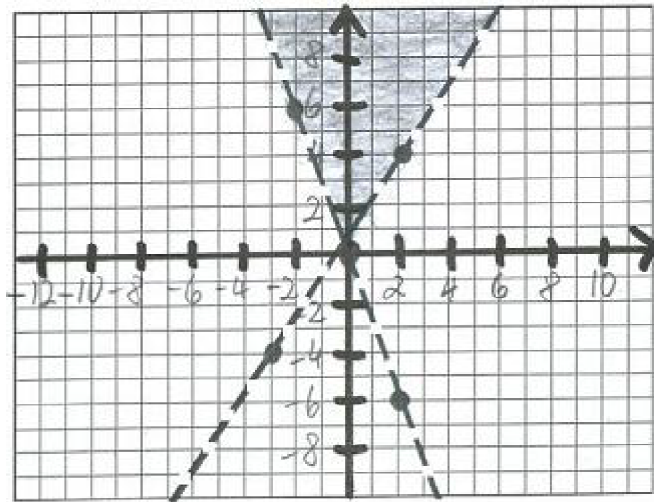
$$y > 2x$$

A system of linear inequalities can only be solved graphically. That is, we graph each inequality on the same pair of axes. The solution set of the system will be the overlapping region of the solution sets of the two inequalities.

We have already graphed the two inequalities separately in Problems 3 and 4 above. Below you can find their graphs one more time.



Now we can determine the solution of the system of these two inequalities by observing where the shaded regions overlap.



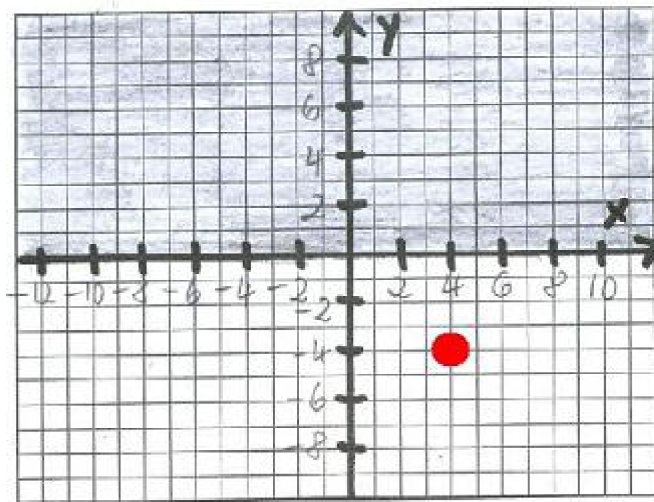
### Problem 8:

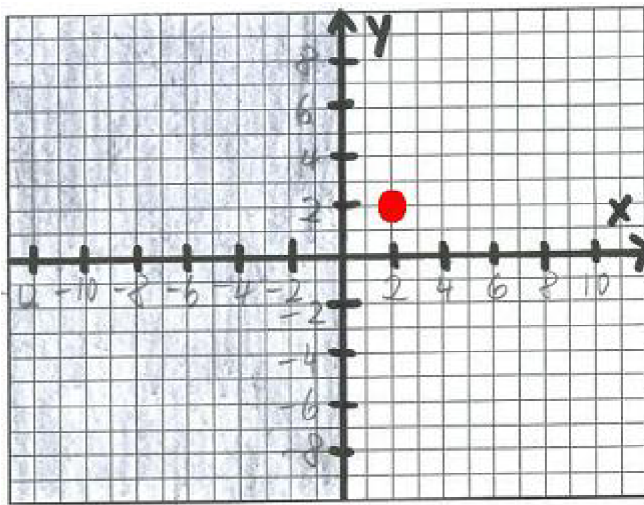
Solve the following system of linear inequalities:

$$y \geq 0$$

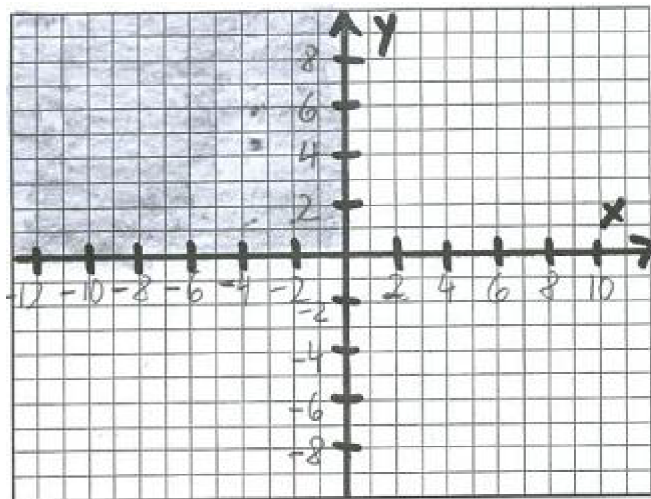
$$x \leq 0$$

We have already graphed the two inequalities separately in Problems 5 and 6 above. Below you can find their graphs one more time.



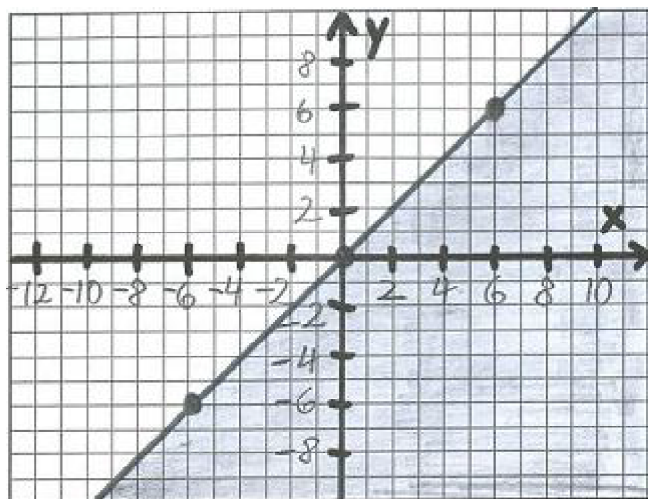


Now we can determine the solution of the system of these two inequalities by observing where the shaded regions overlap.



### Problem 9:

Find the inequality that represents the shaded region.



First, let's find the equation of the boundary line. Since it is solid, we know that we have either a "greater than or equal"  $\geq$  or a "less than or equal"  $\leq$  situation.

We note that the boundary line has one intercept at the origin  $(0, 0)$  and a point at  $(6, 6)$ . This allows us to find the equation as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

First, we find the slope  $m$  given its formula

$$m = \frac{6 - 0}{6 - 0} = \frac{6}{6} = 1$$

Next, we'll use the *point-slope form* of the linear equation

$$y - y_1 = m(x - x_1) \text{ and how about the point } (0, 0)?$$

$$y - 0 = 1(x - 0)$$

then

$$y = x$$

Now we need to find out if the shaded region is represented by  $y \geq x$  or  $y \leq x$ .

First, let's assume it's  $y \geq x$  then a point within the shaded region should make this inequality true.

Let's use  $(1, 0)$  for simplicity's sake. Is  $0 \geq 1$ ? No!

Therefore, the shaded region is represented by  $y \leq x$ .