DETAILED SOLUTIONS AND CONCEPTS - THE LAWS OF EXPONENTS

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PLEASE NOTE THAT YOU MUST BE ABLE TO DO THE FOLLOWING PROBLEMS WITHOUT A CALCULATOR!

The Laws of Exponents

Please be aware that the letters *a*, *b*, *m* , and *n* are replacements for any real number. However, when the letters are identical, we must use the SAME number replacement!

$$a^m a^n = a^{m+n}$$

When an exponential expression is multiplied by another exponential expression **having the same base**, the powers are added.

$$a^0 = 1,\ a \neq 0$$

Any number, except for 0, raised to the zero power results in a value of 1.

$$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$

When a number is raised to a negative power, the exponential expression can be placed in the denominator of a fraction with numerator 1, but the negative sign in the exponent changes to a positive sign.

$$\frac{a^m}{a^n} = a^{m-n}$$

When an exponential expression is divided by another exponential expression **having the same base**, the power in the denominator is subtracted from the power in the numerator.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

When a fraction is raised to a power, this power can be distributed to the numerator and to the denominator.

$$(ab)^m = a^m b^m$$

When a product is raised to a power, each factor is raised to the power.

$$\left(a^m\right)^n = a^{m(n)}$$

When an exponential expression is raised to a power, the powers are multiplied.

Applications of $a^m a^n = a^{m+n}$. When an exponential expression is multiplied by another exponential expression **having the same base**, the powers are added.

Problem 1:

Multiply $x^2 \cdot x^3$.

$x^2 \cdot x^3 = x^{2+3} = x^5$

Problem 2:

Multiply $x \cdot x^6$

$x \cdot x^6 = x^{1+6} = x^7$

Problem 3:

Multiply $10^2 \cdot 10^5$

$10^2 \cdot 10^5 = 10^{2+5} = 10^7 = 10{,}000{,}000$

NOTE: $10^2 \cdot 10^5 \neq 100^{2+5}$

Problem 4:

Multiply $10^2 \cdot 6^5$

$10^2 \cdot 6^5 = 10^2 \cdot 6^5 = 100 \cdot 7{,}776 = 777{,}600$.

NOTE: $10^2 \cdot 6^5 \neq 60^{2+5}$

Please note that the law tells us that we have to have identical numbers in the base before we can add the exponents.

Problem 5:

Multiply $2(10a)$.

$$\begin{aligned} 2(10a) &= 2(10) \cdot a \\ &= 20a \end{aligned}$$

Problem 6:

Multiply $2xy^2(-3xy^2)$.

When you are multiplying two or more terms containing variables, the operation becomes easier if you group together the numbers and the exponential expressions with like base and like powers as follows:

$$\begin{aligned} 2xy^2(-3xy^2) &= 2(-3)\cdot x\cdot x\cdot y^2\cdot y^2 \\ &= -6x^2y^4 \end{aligned}$$

NOTE: You do not have to write down the "grouping" step. Instead you can write the answer right away.

Problem 7:

Multiply $-2(3a)(-5bc^2)(-2ac)$.

This multiplication becomes easier if we regroup as follows:

$$\begin{aligned} -2(3a)(-5bc^2)(-2ac) &= -2(3)(-5)(-2)\cdot a\cdot a\cdot b\cdot c^2\cdot c \\ &= -60a^2bc^3 \end{aligned}$$

Problem 8:

Simplify $-2a^3b^4(-3a^5b^7)$.

The word "simplify" takes on many meanings in mathematics. Often you must figure out its meaning from the mathematical expression you are asked to "simplify." Here we are asked to "simplify" instead of to multiply.

This multiplication becomes easier if we regroup as follows:

$$\begin{aligned} -2a^3b^4(-3a^5b^7) &= -2(-3)\cdot a^3\cdot a^5\cdot b^4\cdot b^7 \\ &= 6a^8b^{11} \end{aligned}$$

Applications of $a^0 = 1,\ a \neq 0$. Any number, except for 0, raised to the zero power results in a value of 1.

Problem 9:

Find the value of 5^0.

$5^0 = 1$

Problem 10:

Find the value of $1{,}000{,}000^0$.

$1{,}000{,}000^0 = 1$

Problem 11:

Find the values of $(-2)^0$ and -2^0.

$(-2)^0 = 1$

$-2^0 = -1(2^0) = -1(1) = -1$.

By the *Order of Operation*, exponential expressions are simplified **BEFORE** we multiply (in this case by -1)!

Applications of $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$. When a number is raised to a negative power, the exponential expression can be placed in the denominator of a fraction with numerator 1, but the negative sign in the exponent changes to a positive sign.

Problem 12:

Rewrite in terms of positive exponents: y^{-4}

$$y^{-4} = \frac{1}{y^4}$$

Please note that $y^{-4} \neq -y^4$.

Problem 13:

Rewrite in terms of positive exponents: 3^{-3}

$$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

Problem 14:

Rewrite in terms of positive exponents: $(-4)^{-3}$ and -4^{-3}.

$$(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64} \text{ or } -\frac{1}{64}$$

Now observe, $-4^{-3} = -\frac{1}{4^3} = -\frac{1}{64}$.

Here we actually have ***-1(4)⁻³*** and we have to go by the *Order of Operation*.

Problem 15:

Rewrite in terms of positive exponents: $(-4)^{-4}$ and -4^{-4}.

$$(-4)^{-4} = \frac{1}{(-4)^4} = \frac{1}{256}$$

Now observe, $-4^{-4} = -\frac{1}{4^4} = -\frac{1}{256}$

Here we actually have $-1(4)^{-4}$ and we have to go by the *Order of Operation*.

Applications of $\frac{a^m}{a^n} = a^{m-n}$. When an exponential expression is divided by another exponential expression **having the same base**, the power in the denominator is subtracted from the power in the numerator.

Problem 16:

Divide $\frac{x^5}{x^2}$.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

Problem 17:

Divide $\frac{x^6}{x}$

$$\frac{x^6}{x} = x^{6-1} = x^5$$

Problem 18:

Divide $\dfrac{10^5}{10^2}$.

$$\frac{10^5}{10^2} = 10^{5-2} = 10^3 = 1{,}000$$

NOTE: $\dfrac{10^5}{10^2} \neq 1^{5-2}$

Problem 19:

Divide $\dfrac{6^4}{3^2}$.

$$\frac{6^4}{3^2} = \frac{6^4}{3^2} = \frac{1{,}296}{9} = 144$$

NOTE: $\dfrac{6^4}{3^2} \neq 2^{4-2}$

Problem 20:

Divide $\dfrac{x^3}{x^3}$.

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

and $x^0 = 1$

Problem 21:

Divide $\dfrac{x^2}{x^5}$.

$$\frac{x^2}{x^5} = x^{2-5} = x^{-3}$$

or $x^{-3} = \dfrac{1}{x^3}$

Problem 22:

Simplify $\dfrac{-12x^3y^5}{3xy^2}$.

The word "simplify" takes on many meanings in mathematics. Often you must figure out its meaning from the mathematical expression you are asked to "simplify." Here we are asked to "simplify" instead of to divide.

Here we must group together the numbers and the exponential expressions with like base as follows:

$$\frac{-12x^3y^5}{3xy^2} = \frac{-12}{3}\cdot\frac{x^3}{x}\cdot\frac{y^5}{y^2}$$
$$= -4x^2y^3$$

NOTE: You do not have to write down the "grouping" step. Instead you can write the answer right away.

Problem 23:

Simplify $\dfrac{8ab^3}{2}$.

$$\frac{8ab^3}{2} = \frac{8}{2}\cdot\frac{a}{1}\cdot\frac{b^3}{1}$$
$$= 4ab^3$$

Problem 24:

Simplify $\dfrac{4ab^5}{3ab^{-4}}$.

$$\frac{4ab^5}{3ab^{-4}} = \frac{4}{3}\cdot\frac{a}{a}\cdot\frac{b^5}{b^{-4}}$$
$$= \frac{4}{3}\cdot 1\cdot b^{5-(-4)}$$
$$= \frac{4}{3}b^{5+4}$$
$$= \frac{4}{3}b^9 \text{ or } \frac{4b^9}{3}$$

Problem 25:

Simplify $\dfrac{-18x^{3}y^{-9}}{3x^{2}y^{2}}$. Write your answer with positive exponents only!

$$\frac{-18x^{3}y^{-9}}{3x^{2}y^{2}} = \frac{-18}{3}\cdot\frac{x^{3}}{x^{2}}\cdot\frac{y^{-9}}{y^{2}}$$
$$= -6xy^{-9-2}$$
$$= -6xy^{-11}$$
$$= \frac{-6x}{y^{11}}$$

Applications of $\left(\dfrac{a}{b}\right)^{m} = \dfrac{a^{m}}{b^{m}}$. When a fraction is raised to a power, this power can be distributed to the numerator and to the denominator.

Problem 26:

Find the value of $\left(\dfrac{2}{x}\right)^{3}$.

$$\left(\frac{2}{x}\right)^{3} = \frac{2^{3}}{x^{3}} = \frac{8}{x^{3}}$$

Problem 27:

Simplify $\left(\dfrac{5}{2}\right)^{3}$.

The word "simplify" takes on many meanings in mathematics. Often you must figure out its meaning from the mathematical expression you are asked to "simplify." Here we are asked to "simplify" instead of to find the value of the number.

$$\left(\frac{5}{2}\right)^{3} = \frac{5^{3}}{2^{3}} = \frac{125}{8}$$

Problem 28:

Simplify $\left(\frac{1}{3}\right)^4$.

$$\left(\frac{1}{3}\right)^4 = \frac{1^4}{3^4} = \frac{1}{81}$$

Note that the number 1 raised to any power will always have a value of 1.

Problem 29:

Simplify $\left(\frac{-3}{5}\right)^2$.

$$\left(\frac{-3}{5}\right)^2 = \frac{(-3)^2}{5^2} = \frac{9}{25}$$

Applications of $(ab)^m = a^m b^m$. When a product is raised to a power, each factor is raised to the power.

Applications of $\left(a^m\right)^n = a^{m(n)}$. When an exponential expression is raised to a power, the powers are multiplied.

Problem 30:

Find the value of the number $(5x)^2$.

$$(5x)^2 = 5^2 x^2 = 25x^2$$

Please note that $(5+x)^2 \neq 5^2 + x^2$ and $(5-x)^2 \neq 5^2 - x^2$. Later on we will learn how to deal with sums and differences raised to a power!

Problem 31:

Simplify $(x^2)^3$.

The word "simplify" takes on many meanings in mathematics. Often you must figure out its meaning from the mathematical expression you are asked to "simplify." Here we are asked to "simplify" instead of to find the value of the number

$$(x^2)^3 = x^{2(3)} = X^6$$

Note that $x^2 \cdot x^3 = x^{2+3} = x^5$.

Problem 32:

Simplify $(-3a^3b^4c)^2$.

$$\begin{aligned}(-3a^3b^4c)^2 &= (-3)^2(a^3)^2(b^4)^2(c)^2 \\ &= 9a^6b^8c^2\end{aligned}$$

Please note that this law extends to any product containing infinitely many factors.

Problem 33:

Simplify $(-5h^{-1}k^{-2})^{-3}$.

$$(-5)^{-3}(h^{-1})^{-3}(k^{-2})^{-3}$$

and $-\dfrac{1}{5^3}h^3k^6 = -\dfrac{1}{125}h^3k^6$ or $-\dfrac{h^3k^6}{125}$